Robust Control for Irrigation Canals

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Abstract— **Modern controllers are based on the use of mathematical models. However the models are always obtained through a reduction in the complexity of reality. Consequently, their ability to properly represent the general behavior of the processes is very limited. Therefore, it is advantageous to analyze the problem resulting from model uncertainty in the control of irrigation main canal pools. This problem has often been ignored in theoretical studies and in practical process control. This paper will first show a principal gains method on how to achieve the benefit of feedback in the face of uncertainties. Then it will present irrigation main canal pools example which illustrates the use of robust control to provide satisfactory performance despite of irrigation main canal pools.**

Index Terms— Irrigation canal; Robust control; Principal gains;

I. INTRODUCTION

The standard modern approach, to process control consists in constructing a mathematical model of the process and then using explicitly this model in the controller. However, there are two major problems with this approach: first, the model is only a simplified representation of the process which is generally much more complex; second, the process behavior continuously changes. For these two reasons there is inevitably a mismatch between the plant and the model. Such model uncertainties are responsible for the degradation of the controller. Hence, the first step in a robust control study is to quantify these uncertainties. For that purpose, an irrigation main canal pools is used, experiments based on the response to a step-like input were carried out at the first pool in order to obtain a linear mathematical model with which to describe its dynamic behavior [1]. The case of study presented in this paper is about the first pool of the Aragon Imperial Main Canal (AIMC) that exhibits large variations in its characteristic parameters [3]. The dynamic behavior of the first pool can be represented by second order model with a time delay [2] and by varying the operating

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conditions. The models for four operating conditions different from the nominal conditions are obtained and their multiplicative uncertainties are also determined. In the next step a robust controller with principal gains method is obtained for irrigation main canal pools.

This paper is organized as follows. Section II presents preliminaries and robustness condition, Section III introduces the principal gains method, Section IV introduces the main irrigation channel pool and application of the proposed control scheme and the controller, where the time domain and frequency domain tunings are also developed. , in this section we compare the robustness of controllers. Finally, Section V resumes the main conclusions obtained during the development of this work.

II. PRELIMINARIES

It is necessary to recall the basic required performances of a control loop in the frequency domain. Figure.1 shows the classical structure of a control loop with the main components: the controller (transfer matrix K(s)), the process uncertainty at the process output $\Delta_m(s)$, the set-point r, the loop's error e and finally the manipulated variable u and the output y. Let G'(s) the transfer matrix of the true plant, all perturbed regimes, and then the following relation can be written:

$$G'(s) = [I + \Delta m(s)]G(s) \tag{1}$$

The largest singular value of $\Delta m(s)$ is obtained from (1):

$$\sigma_{\max}\left([\Delta m(s)]\right) = \sigma_{\max}([G'(s) - G(s)]G^{-1}(s)) \tag{2}$$
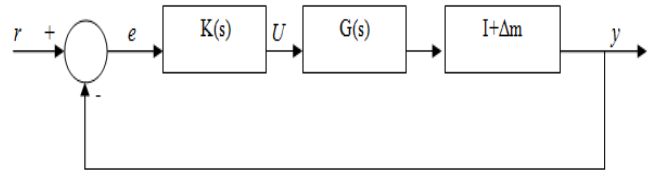

Fig. 1. Feedback configuration with multiplicative uncertainties.

A. Robust stability

Assume that the nominal feedback system G(s) (i.e. with $\Delta_m(s) = 0$) is stable, then the true feedback system G'(s) is stable if the following inequality holds [4]:

$$\sigma_{\max}[T(s)] < \frac{1}{\sigma_{\max}[W_t(s)]} \tag{3}$$

Where T(s) is the nominal closed loop transfer matrix given by:

$$T(s) = G(s)K(s)[I + G(s)K(s)]^{-1} \qquad (4)$$

And $W_t(s)$ is a stability specification matrix such as:

$$\sigma_{max}[\Delta m(s)] \leq \sigma_{max}[W_t(s)] \qquad (5)$$

Then σ_{max} [T(s)], the largest singular value of the nominal closed loop transfer matrix is a reliable indicator of the robust stability of the feedback system. Then the robustness condition of the feedback system is given by (3).

B. Robust Performances

Let $W_p(s)$ a performance specification matrix, weighting matrix, then the robust performances of all perturbed regimes $G'(s)$ are satisfied if the following inequality holds [4]-[5]:

$$\sigma_{max}[S(s)] \leq \frac{1}{\sigma_{max}[W_p(s)]} \qquad (6)$$

Where S(s) is the sensitivity matrix given by:

$$S(s) = [I + G(s)K(s)]^{-1} \qquad (7)$$

In fact, the largest singular value of the sensitivity matrix $\sigma_{max}(s)$ [7], [8] and [9] is also an indicator of the sensitivity of the system response to a change of the plant character. In conclusion, the inequalities (3) and (6) represent the robustness conditions and must be satisfied to obtain a robust controller.

III. PRINCIPAL GAINS METHOD

The principal gains method is based on finding a controller with the following structure [6]:

$$K(s) = K1 * K2(s) * K3(s) * K4(s) \qquad (8)$$

Where: $K1 = G^{-1}(0)$ is the inverse static gain. It is used to decouple the process in low frequency.

$K2(s) = \dfrac{1}{s}$ Is a set of integrators to eliminate the static error.

K3 is a compromise coefficient between the stability and performances.

K4(s) is a structure to reduce the resonance magnitude in middle and high frequency. In order to not affect the controller in low frequency, we have to set $K4(0) = I$, this can be obtained by minimization of the following criteria [4]:

$$\min K4(J) = \min K4 \max w [\sigma_{max}(T) \sigma_{max}(\Delta m)] \qquad (9)$$

Where: $\sigma_{max}(T) * \sigma_{max}(\Delta m)$ is a stability robust condition.

IV. APPLICATION: IRRIGATION MAIN CANAL POOLS

A. Irrigation main canal description [1]

The irrigation main canal considered in this paper is the Aragon Imperial Main Canal (AIMC), which obtains its water from the Ebro River [2]. The AIMC is a 108 km long cross-structure canal with a design head discharge of $30m^3$/s. It has a trapezoidal cross-section and ten pools of different lengths which are separated by undershoot flow gates.

The representation of this canal is given in Fig. 2, where the manipulated variable $u_1(t)$ is the upstream gate position, and the output $y_1(t)$ is the downstream end water level.
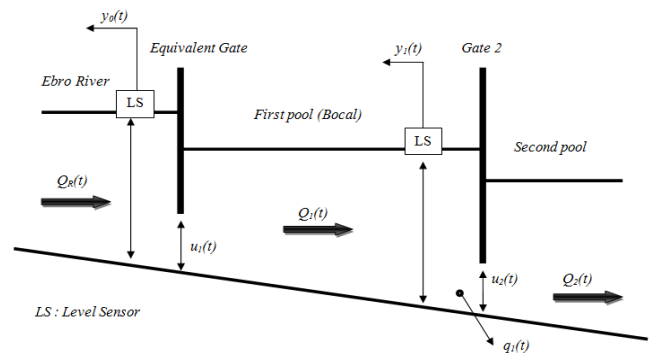

Fig. 2. Equivalent schematic representation of the irrigation main canal pool.

A dynamic process model was developed with the aid of step responses. The transfer function of the nominal regime of the irrigation main canal pools is given by [3]:

$$G(s) = \frac{\Delta y_1(s)}{\Delta u_1(s)} = \frac{K}{(1 + T_1(s))(1 + T_2(s))} e^{-\tau s}$$

The models for four operating conditions different from the nominal conditions (perturbed regimes) are also obtained with the aid of step responses [1].

$$G(s) = \frac{\Delta y_1(s)}{\Delta u_1(s)} = \frac{0.01}{(1 + 500s)(1 + 300s)} e^{-360s}$$

$$G(s) = \frac{\Delta y_1(s)}{\Delta u_1(s)} = \frac{0.01}{(1 + 15000s)(1 + 300s)} e^{-360s}$$

$$G(s) = \frac{\Delta y_1(s)}{\Delta u_1(s)} = \frac{0.07}{(1 + 500s)(1 + 300s)} e^{-360s}$$

$$G(s) = \frac{\Delta y_1(s)}{\Delta u_1(s)} = \frac{0.06}{(1 + 1500s)(1 + 300s)} e^{-360s}$$

B. Evaluation of multiplicative uncertainties $\Delta_m(s)$

The largest singular values of the multiplicative uncertainties $\Delta_m(s)$ are determined from (2). These model parameter variations are caused by the following factors: the discharge regime variations through the upstream gates in the operation range, originate changes in pools storage volume between the null discharge volume and the maximum discharge volume in the downstream water levels and in the flow propagation. The result is given in Fig.3, where it is verified that the maximum singular values of these uncertainties are less than one at low frequencies and increase at high frequencies [4].

C. Robustness conditions

From the obtained results (fig. 3) and using (5), the stability specification $W_t(s)$ is represented as follow:

$$W_t(s) = 0.9(1 + 5000s) \qquad (10)$$

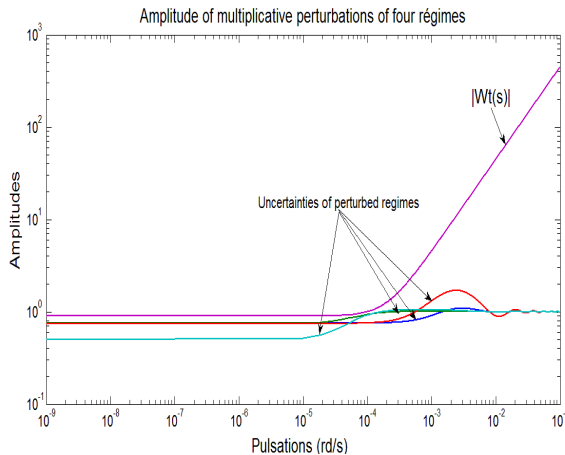Then, the condition of stability robustness is given by inequality (3).

Fig. 3. multiplicative uncertainties $\Delta_m(s)$

The performance specifications for all possible perturbed regimes are defined such that these regimes have the same response time that nominal regime. Then the performance specification $W_p(s)$ is given by:

$$W_p = \frac{(1 + 30000s)}{30000s}$$

The condition for robust performance is given by (6). Finally, the robustness conditions for irrigation main canal pools are represented in fig. 4:
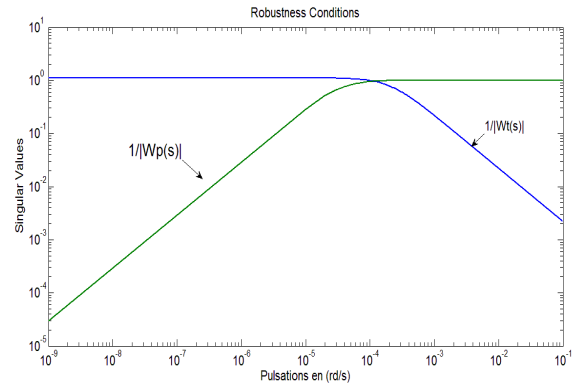

Fig. 4. Robustness conditions.

D. Robust controller with principal gains method

The principal gains method consists of finding a controller $K(s)$ given by (8) such that (9) is satisfied and the conditions (3) and (6) for robust stability and performance are also verified. A simplified model from the nominal regime used in the design is defined by:

$$G_{sn} = \frac{0.0401}{(1 + 880.79s)(1 + 81.27s)}$$

The controller is obtained as:

$$K1 = G^{-1}(0) = \frac{1}{0.0401} = 24{,}9377; \quad K2(s) = \frac{1}{s}$$

The coefficient value (K3 = 0.0011) is obtained by simulation. The structure K4(s) is chosen as:

$$K4(s) = 1 + \alpha s + \beta s^2$$

Where α and β are determined by minimization of criteria (9),

$\alpha = 962.06; \beta = 71581.80$.

The final controller is then given by:

$$K(s) = 0.0274 \frac{(1 + 962.06s + 71581..80s^2)}{s}$$

The results in frequency domain are given in fig. 5.
It is showed that the robustness conditions are not violated, since for multi-variables and mono-variable systems, the stability is guaranteed if the largest singular value of closed loop transfer matrix function $\sigma_{\max}[(\mathrm{T}(s)]$ is lower than the upper bound of the largest singular value of the model uncertainties $\left[\sigma_{\max}[(W_t(s)]\right]^{-1}$. The same idea is used for the robust performance criterion.
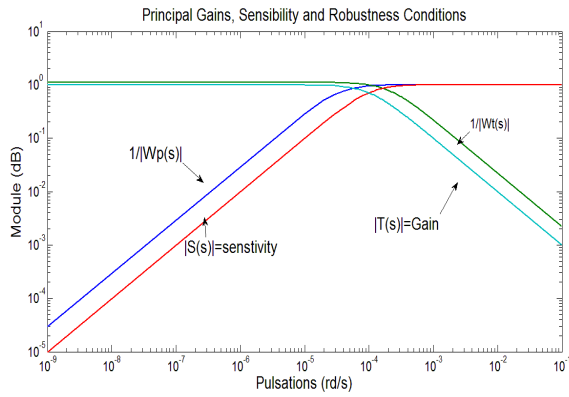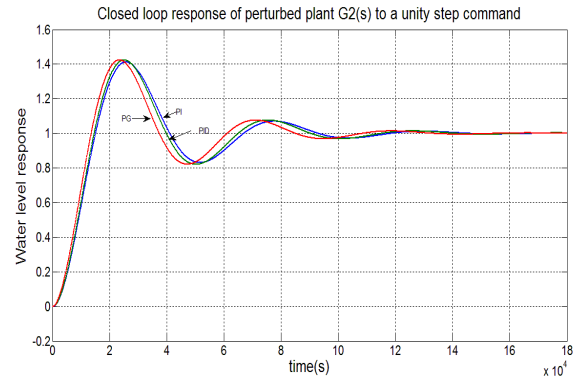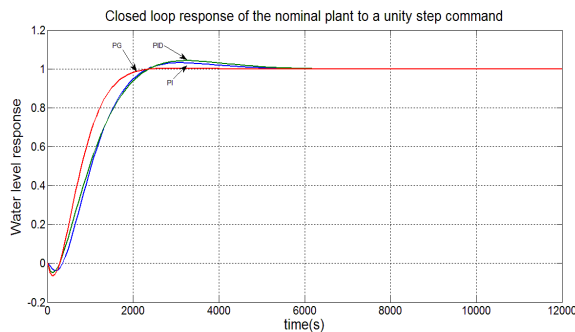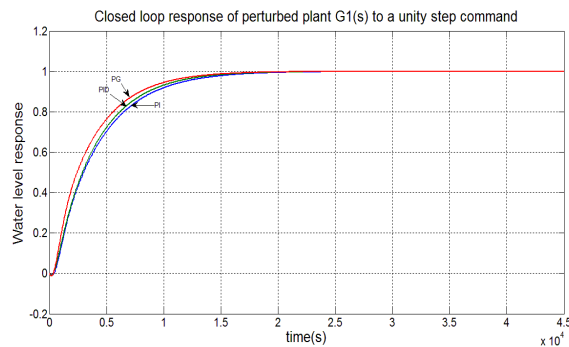
Fig. 5. Frequency results.

(c) Closed loop responses of the perturbed plant $G_2(s)$ with k=0.01: (T1=15000, T2=300, τ=360).
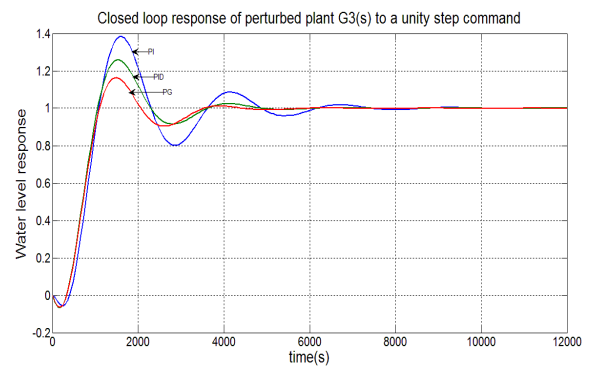
The results in time domain are represented in fig. 6.

The simulation $\left[e^{-360s} = (1-180s)/(1+180s)\right]$ have been carried out using Padé approximation, where the sampling period is set T=60s. The stability of all regimes, a good performance and a fast response time with principal gains controller (PG) are observed. It is noted that our obtained results are very encouraging with the PI and PID controllers reported by [1].
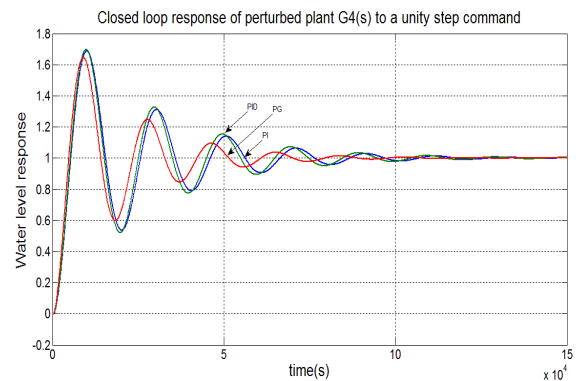

(d) Closed loop responses of the perturbed plant $G_3(s)$ with k=0.07: (T1=500, T2=300, τ=360).

(a) Closed loop of the nominal plant G(s) to a unity step command.

(e) Closed loop responses of the perturbed plant $G_4(s)$ with k=0.06: (T1=15000, T2=300, τ=360).

Fig. 6. (a) (b) (c) (d) (e) Step responses of nominal of and perturbed regimes of irrigation canal.

(b) Closed loop responses of the perturbed plant $G_1(s)$ with k=0.01: (T1=500, T2=300, τ=360).

V. Conclusion

In this paper a principal gains method to achieve the benefit of feedback in the face of uncertainties has been investigated and successfully applied at irrigation main canal pools.

The results are very encouraging since control-oriented models facilitate the design of high-performance robust controllers, which allow the operability and efficiency of irrigation main canal pools to be increased and service to the users to be improved.

The theory behind the robust control tools is simplified to be easily transmitted to irrigation processing students and engineers. Works are under progress to investigate the principal gains and H infinity methods for multi pools of irrigation main canal.

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