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Fuzzy sliding mode observer design for anaerobic digestion process

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Abstract**—** This work, deals with the design of fuzzy sliding mode observers (FSMO) where fuzzy modelling and state estimation/observation are powerful methods for the control and diagnosis of very complex nonlinear systems. Especially where the system's states are not all available for measurement because hardware sensor measurements do not exist. This paper presents, some results obtained from the design of fuzzy sliding mode observers for an anaerobic digestion system represented by T-S fuzzy model.

Keywords**— T-S fuzzy modelling, state estimation, Sliding mode, Linear Matrix Inequalities, Anaerobic digester.**

I. INTRODUCTION

Anaerobic digestion process is a waste treatment technology which allows for the mineralisation of the organic material through the controlled destruction of the organic waste to produce a digestate product that can be used as a fertilizer or soil conditioner and it produces renewable energy. Wastewater treatment plants (WWTP) contribution is reducing the impact of human waste on the environment. This is accomplished by removing most of the oxygen demand caused by chemical and organic wastes in the wastewater prior to its return to the environment [1].

Moreover, as a renewable energy source, anaerobic digestion plant (ADP) has great potential. It is designed to encourage the growth of anaerobic bacteria, specially the methane producing bacteria that decrease organic solids by reducing them to soluble substances and gases mostly a mixture of carbon dioxide and methane. This biogas can be used directly as cooking fuel, in combined heat and power gas engines are upgraded to natural gas quality biomethane. The utilisation of biogas as a fuel can replace the fossil fuels.

The bioprocesses are complexes, nonlinear and uncertain systems. They are in most of the time, badly definite, and submitted to unexpected changes.

Often, it is more useful to represent nonlinear complex systems by the aggregation of local linear time invariant (LTI) models obtained by linearization around multiple operating points. Fuzzy systems have the capacity to handle in the same framework numeric and linguistic information. This characteristic made these systems very useful to handle expert control tasks [4].

Takagi-Sugeno modeling is always proving the effectiveness to represent nonlinear systems because they permit to represent most of nonlinear systems, whatever is its complexity, by a simple “IF…THEN” structure. The fuzzy representation introduced in [4] constitutes an interesting alternative in the domain of control, observation and diagnosis of nonlinear systems.

Anaerobic digestion variables are not all available for measurement because hardware sensor measurements does not exist which are very important for control and diagnosis [5]. In this work, we used sliding mode observers as soft sensors to estimate all variables of ADP.

Sliding mode observers (SMO) and unknown input observers are a developed type of observer based method [15], [16], [17]. The structure of SMOs is the key of their robustness and insensitivity to the various types of uncertainties discussed in [13], [12]. These observers are more strong than Luenberger observer because they offer advantages inherent robustness by taking in consideration the modelling uncertainties, faults, perturbations or/and non linearities in the plant [11]. To ensure the convergence of the state estimation error, an LMI based design procedures for different types of observers are constructed in [8], [9], [10], [17] and [19]. In this paper, we investigate the use of fuzzy sliding mode observer for anaerobic digestion system state estimation

In Section II, description and model structure of anaerobic digestion system. In Section III, an overview of T-S fuzzy modelling. In Section IV, outlines the T-S fuzzy sliding mode observer design and convergence conditions of the FSMO presented in terms of linear matrix inequality (LMI) formulation. Experimental results are presented in Section V. Finally, some concluding remarks as well as some possible improvements are given in Section VI.

II. DESCRIPTION OF ANAEROBIC DIGESTION PROCESS

Anaerobic digestion (ADP) is a multi-step biological treatment process in which bacteria break down organic material consist of proteins, carbohydrates, and lipids, which are converted to biogas in the absence of oxygen.
The organic solids are degraded through four sequential, metabolic stages shown in fig.1: 1.) Organic solids are broken down by enzymes, 2.) Acidogenic fermentations are most important, acetate is the main end product. Volatile fatty acids are also produced along with carbon dioxide and hydrogen. 3.) Breakdown of volatile acids to acetate and hydrogen. 4.) Acetate, formaldehyde, hydrogen and carbon dioxide are converted to methane and water. There are several models available for the simulation of the anaerobic digestion of wastewater solids and sludges. Some of these models focus on a specific component of anaerobic digestion, such as microbial kinetics, while others attempt to encompass the overall digestion process [1], [2].

$$\begin{cases} \dot{X}_1 = (\mu_1 - \alpha D) X_1 \\ \dot{X}_2 = (\mu_2 - \alpha D) X_2 \\ \dot{S}_1 = D\left(S_1^{in} - S_1\right) - k_1\mu_1 X_1 \\ \dot{S}_2 = D\left(S_2^{in} - S_2\right) - k_2\mu_1 X_1 - k_3\mu_2 X_2 \\ \dot{Z} = D\left(Z^{in} - Z\right) \\ \dot{C}_{TI} = D\left(C_{TI}^{in} - C_{TI}\right) + k_7\left(k_8 P_{co_2} + Z - C_{TI} - S_2\right) + \\ k_4\mu_1 X_1 + k_5\mu_2 X_2 \\ Q_{CH_4} = k_6\mu_2 X_2 \end{cases}$$
(1)

Where { X_1, X_2, S_1, S_2, Z, C_{TI} } are respectively, the bacteria concentration acidogenic (g/L), methanogenic bacteria (g/L), soluble COD (g/L), total volatile fatty acids VFA (mmol/L), total inorganic carbon and total alkalinity (meq/L). For these variables " ^{in} " indicates the influent concentration. D is the dilution rate and Q_{CH_4} is the biogas (methane) flow rate (L/h).
Nonlinear functions μ_1 and μ_2 represent the Haldane growth rates and have the following structure

$$\mu_1 = \mu_{\max 1}\frac{S_1}{k_{s_1} + S_1} \text{ and } \mu_2 = \mu_0 \frac{S_2}{k_{s_2} + S_2 + \left(\frac{S_2}{k_{12}}\right)^2}$$
(2)

Where $k_1, ..., k_8$ are the yield coefficients. The partial pressure P_{co_2} of CO_2 have the following structure:

$$P_{CO_2} = \frac{\Phi - \sqrt{\Phi^2 - 4k_8 P_r [CO_2]}}{2k_8}$$
(3)

Where P_r is total pressure in the reactor (atm).

$$\Phi = k_8 P_r + [CO_2] + \frac{k_6\mu_2 X_2}{k_1}$$
and
$$[CO_2] = C_{TI} + S_2 - Z$$
(4)

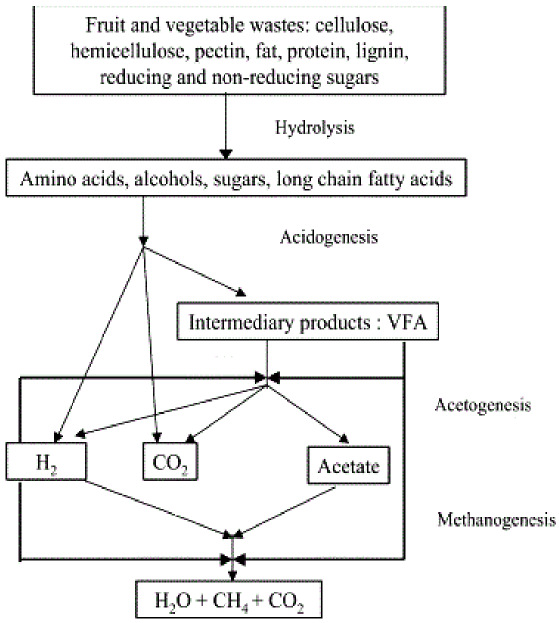

Fig 1. The general schematic diagram of anaerobic digestion process

III. FUZZY MODELLING

A. *Linearization of the process model*

Dynamical nonlinear uncertain systems Eq. 1 can be described by differential equations:

$$\begin{cases}\dot{x}(t)=f(x,u)\\ y(t)=h(x,u)\end{cases}$$
(5)

Where f and h are nonlinear functions.

It is better to represent nonlinear systems by local LTI models Eq. 7 obtained by a multiple operating point linearization. Fuzzy systems have the capacity to handle in the same framework numeric and linguistic information. The obtained linearized model corresponds to the relationship between the variation of the system output and the variation of the system input around this operating point. If the system is linearized around an operating point $(\bar{x}_i,\bar{u}_i)$, the linearized model corresponds to the relationship between the variations of the system states x and inputs u such that:

$x=x-\bar{x}_i$ and $u=u-\bar{u}_i$

$$A_i=\left.\frac{\partial f}{\partial x}\right|_{(\bar{x}_i,\bar{u}_i)}$$
$$B_i=\left.\frac{\partial f}{\partial x}\right|_{(\bar{x}_i,\bar{u}_i)}$$
$$C_i=\left.\frac{\partial h}{\partial x}\right|_{(\bar{x}_i,\bar{u}_i)}$$
(6)

From Eq.6 we obtain the following local state-space description:

$$\begin{cases}\dot{x}(t)=A_ix(t)+B_iu(t)+\mathrm{a}_i\\ y(t)=C_ix(t)+c_i\end{cases}$$
(7)

There is a possibility to obtain the local state space model from experimental data [20].

B. T-S Fuzzy model

Fuzzy systems are used to simplify the complexity of strongly nonlinear systems. This representation belongs to the paradigm of behavioral representation in opposition to the structural representation (neural networks). The foundation of this paradigm is that intelligent behavior can be obtained by the use of the structures that not necessarily resemble the human brain. The model of the plant is assumed to be given by the T-S models, where the i-th rule is of the form:

Rule i:

IF z_1 is M_{1i}, AND,…, AND, z_g is M_{gi}

THEN $\begin{cases}\dot{x}(t)=A_ix(t)+B_iu(t)+E_id(t)+\mathrm{a}_i\\ y(t)=C_ix(t)+c_i\end{cases}$ where

$x\in R^n$ is the state vector, $u\in R^m$ is the control input vector, $y\in R^p$ is the system output vector and $d\in R^q$ contains unknown inputs and it is assumed to be bounded Eq. 6. z (t) = [z_1 (t),..., z_g (t)] ^{T} is the decision vector, M_{gi} are membership functions, $A_i\in R^{n\times n}$, $B_i\in R^{n\times m}$, $C_i\in R^{p\times n}$, $E_i\in R^{n\times q}$ are known matrices and i = {1,...,r} with r is the number of rules.

$$\|d(t)\|\le\rho$$
(8)

Such that B_i is full rank, the pairs (A_i,B_i) are completely controllable and the pairs (A_i,C_i) are completely observable.

The whole dynamic system defined by T-S fuzzy system is given by:

$$\begin{cases}\dot{x}(t)=\sum_{i=1}^{r}h_i(\mathrm{z}(t))\left[A_ix(t)+B_iu(t)+E_id(t)+a_i\right]\\ y(t)=\sum_{i=1}^{r}h_i(\mathrm{z}(t))\left[C_ix(t)+c_i\right]\end{cases}$$
(9)

$$h_i(\mathrm{z}(t))=\frac{\mu_i(\mathrm{z}(t))}{\sum_{i=1}^{r}\mu_i(\mathrm{z}(t))}$$

Where

$\mu_i(\mathrm{z}(t))=\prod_{j=1}^{g}\mu_{ij}(\mathrm{z}_j(t))$ is the degree of fulfilment of the i-th rule, $\mu_{ij}(\mathrm{z}_j(t))$ is the grade of membership function of $z_j(t)$ in M_{ij}, and M_{ij} is a fuzzy sets.

$h_i(\mathrm{z}(t))$ must satisfy the following constraints:

$$\begin{cases}\sum_{i=1}^{r}h_i(\mathrm{z}(t))=1,\\ h_i(\mathrm{z}(t))\in[0,1],\forall i=1,...,r\end{cases}$$

IV. FUZZY SLIDING MODE OBSERVER (FSMO) DESIGN

Sliding mode observer offer advantages inherent robustness by taking in consideration the modelling uncertainties, perturbations or/and non linearities in the plant [19].

IF Z_1 is M_{1i}, AND,…, AND, Z_g is M_{gi}

$$\begin{cases}\dot{\hat{x}}(t)=A_i\hat{x}(t)+B_iu(t)+L_ie_y(t)+\varphi_i(t)\\ \hat{y}(\mathrm{t})=\mathrm{C}_i\,\hat{x}(t)\end{cases}$$

THEN

The whole FSMO design is:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(z(t))\left[A_i\hat{x}(t) + B_i u(t) + L_i e_y(t) + \varphi_i(t)\right] \\ \hat{y}(t) = \sum_{i=1}^{r} h_i(z(t))C_i\,\hat{x}(t) \end{cases}$$

(10)

Where $h_i(z(t))$ are the same as the weights in the T-S fuzzy model Eq. 9, $L_i \in R^{n\times p}$ is the observer gain matrix and $\varphi_i \in R^m$ is an external discontinuous vector of sliding mode observer to compensate the errors due the unknown inputs, such that $C_i L_i$ is a non singular matrix.

The aim of the observer design task is to find appropriate gain matrices L_i and variable vectors that guarantee the asymptotic convergence of $\hat{x}(t)$ toward $x(t)$.

To analyze the convergence of the SMO, the state reconstruction error is defined as follows:

$$e_x(t) = x(t) - \hat{x}(t)$$

(11)

$$e_y(t) = y(t) - \hat{y}(t) = \sum_{i=1}^{r} h_i(z(t))(C_i e_x(t))$$

(12)

Using Eq. 11 and Eq. 12 we got:

$$\dot{e}_x(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))(\bar{A}_{ij}e(t) + E_i d(t) - \varphi_i(t))$$

(13)

Where

$\bar{A}_{ij} = A_i - K_i C_j$ for i,j = 1,…,r and i < j.

The asymptotic convergence of the estimation error is expressed in the following result:

Theorem 1: Let us consider the T-S fuzzy model Eq. 9, if there exist symmetric and positive definite matrices $P \in R^n$ and $N_i \in R^{n\times p}$, positive scalars ε_1, ε_2 and, the observer's synthesis takes place by the resolution of a set of linear matrix inequalities LMIs and structural constraints given by:

$$\begin{bmatrix} A_i P + PA_i - C_j^T N_i^T - N_i C_j + \gamma I & P \\ P & -\varepsilon_1 I \end{bmatrix} < 0$$

(14)

Then, if the equations $\varphi_i(t)$ are given by the following form:

$$\varphi_i(t) \equiv \begin{cases} \rho^2 \varepsilon_2^{-1} \dfrac{\|PE_i\|^2}{2e_y^T e_y} P^{-1} \sum_{i=1}^{r} h_i C_j^T e_y \ldots if \ldots e_y(t) \neq 0 \\ o\ldots\ldots Otherwise \end{cases}$$

(15)

The fuzzy sliding mode observer Eq. 10 with $L_i = P^{-1}N_i$, guarantees the asymptotic convergence to zero of the estimation error.

Lemma 1: for any matrices X and Y with appropriate dimensions, the following property holds for any positive scalar β :

$$X^T Y + Y^T X \leq \beta X^T X + \frac{1}{\beta} Y^T Y$$

(16)

Proof: Considering the quadratic Lyapunov function:

$$V(e_x) = e_x^T P e_x$$

(17)

Its time derivative:

$$\dot{V}(e_x) = e_x^T P \dot{e}_x + \dot{e}_x^T P e_x$$

(18)

Using Eq. 11, Eq. 13, and Eq. 14 has the form:

$$\dot{V} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)(e_x^T(\bar{A}_{ij}^T P + P\bar{A}_{ij})e_x + 2e_x^T PE_i d - 2e_x^T P\varphi_i)$$

(19)

Using Lemma 1 the derivative of the lyapunov function can be written:

$$\dot{V} \leq \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)(e_x^T(\bar{A}_{ij}^T P + P\bar{A}_{ij})e_x + \varepsilon_1^{-1}e_x^T P^2 e_x) + 2e_x^T PE_i d - 2e_x^T P\varphi_i)$$

(20)

$$\leq \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)(e_x^T(\bar{A}_{ij}^T P + P\bar{A}_{ij} + \varepsilon_1^{-1}P^2)e_x + 2e_x^T PE_i d - 2e_x^T P\varphi_i)$$

(21)

Where

$$\forall\, \mathrm{i,j} \in \{1,\ldots,r\} \,/\, h_i(z(t))h_j(z(t)) \neq 0$$

Applying Lemma 1 for each term of $\dot{V}$ we got:

$$\begin{aligned} 2e_x^T PE_i d &= e_x^T PE_i d + d^T E_i^T P e_x \\ 2e_x^T PE_i d &\leq \varepsilon_2 e_x^T e_x + \varepsilon_2^{-1}\|PE_i d\|^2 \\ 2e_x^T PE_i d &\leq \varepsilon_2 e_x^T e_x + \rho^2\varepsilon_2^{-1}\|PE_i\|^2 \end{aligned}$$

(22)

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$$2e_x^T P\varphi_i = \rho^2 \varepsilon_2^{-1} \frac{\|PE_i\|^2}{e_y^T e_y} e_y^T P P^{-1} \sum_{j=1}^{r} h_j C_j^T e_y$$

$$2e_x^T P\varphi_i = \rho^2 \varepsilon_2^{-1} \|PE_i\|^2$$

(23)

From Eq. 22 and Eq. 23we deduce that:

$$\dot{V} \le \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z) h_j(z) e_x^T (\bar{A}_{ij}^T P + P\bar{A}_{ij} + \varepsilon_1^{-1} P^2 + \varepsilon_2 I) e_x$$

(24)

We assume that, the estimation error is insensitive to the uncertainties modelled by the term E_i d(t) and converges towards zero if the relation Eq. 24 holds.

V. SIMULATION AND RESULTS

In this work, the ADP nonlinear model Eq.1 is approximated by T-S fuzzy models with four state space sub-systems for which local linear fuzzy observers are constructed, by taking [D, Q_{CH_4}] as premise variables and they are fuzzified as shown in figure Fig. 3.

Where [D, S_1^{in} , S_2^{in} , C_{TI}^{in} , Z^{in}] are selected as inputs,

d = [S_1^{in} , S_2^{in} , C_{TI}^{in} , Z^{in}] is the disturbances vector and

[S_1 , S_2 , Z, Q_{CH_4}] are selected as outputs. Using a time evolution of the measured and unmeasured inputs over 40 days shown in Fig. 2, and Fig. 4 represents the biogas output. Fig. 5 illustrates the grade of membership functions.

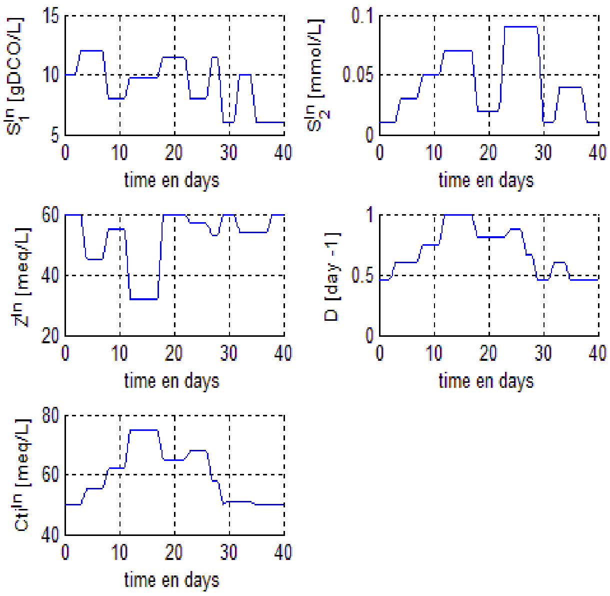

Fig 2. Data sequences of the process.

The use of linearized anaerobic digestion system approximated by T-S fuzzy multiple model

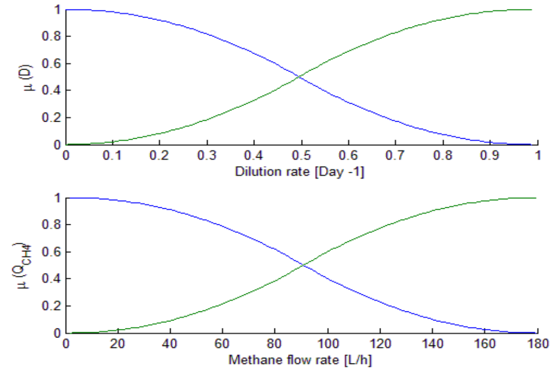

Fig 3. Membership functions for D and Q_{CH_4} .

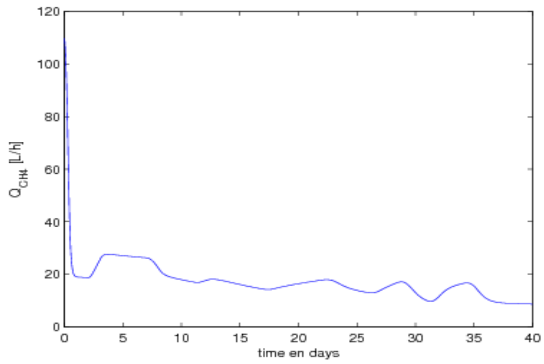

Fig4. Methane flow rate.

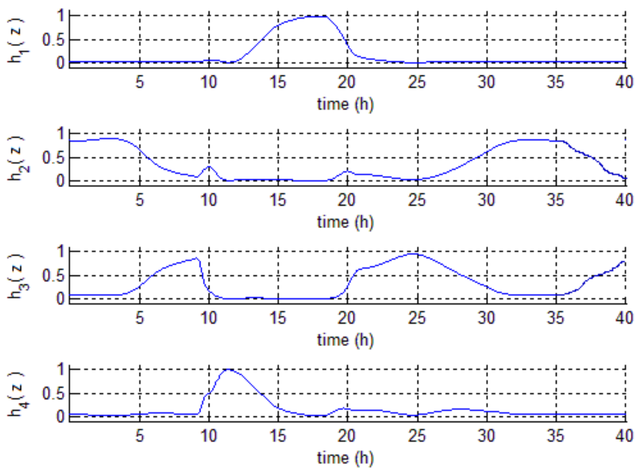

Fig 5. The grade of membership functions

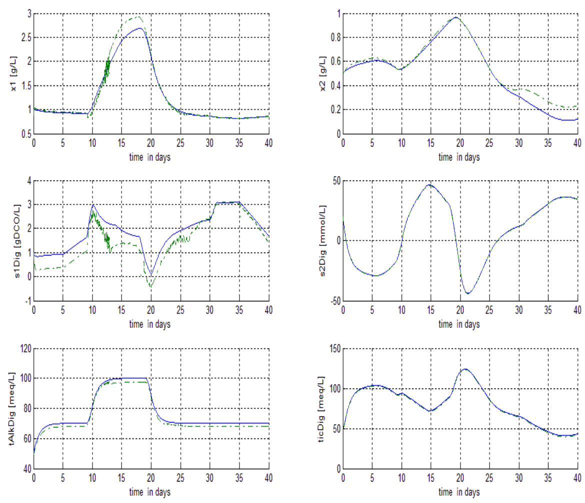

Fig 6. State estimation for ADP using FSMO.

The high performances of Takagi-sugeno fuzzy sliding mode observer for anarerobic digestion process state estimation is shown in Fig. 6.

VI. CONCLUSIONS

The performance of Takagi -Sugeno fuzzy sliding mode observer as a soft sensor for anaerobic digestion system state estimation is proved by simulation results. The ADP behavior has been approximated by T-S fuzzy models. The FSMO can have guaranteed stability and performance and will converge asymptotically to the real system states. The design methodology demands the solution of linear matrix inequalities LMIs using an efficient numerical method.

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