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Index Stock Options Pricing Under Kou Jump-Diffusion Model: SP500 Study

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***Abstract:*The present research points towards the empirical validation of three options valuation models, the ad-hoc Black-Scholes model as proposed by Berkowitz (2001), the constant elasticity of variance model of Cox and Ross (1976) and the Kou jump-diffusion model (2002). Our empirical analysis has been conducted on a sample of 12,499 options written on the SP 500 index that were negotiated during the year 2007 just before the sub-prime crisis. We start by presenting the theoretical foundations of the models of interest. Then we use the technique of nonlinear least squares to estimate the structural parameters of these models from cross-section of option prices. The empirical analysis shows the superiority of the Kou jump-diffusion model which arises from itsability to portray the behavior of market participants and to be closest to the true distribution that characterizes the evolution of these indices. Indeed the double-exponential distribution covers three interesting properties that are: the leptokurtic feature, the memory less property and the psychological aspect of market participants as numerous empirical studies have shown that markets tend to have both overreaction and under reaction over good and bad news respectively.**

***Keywords:*Stock Index Options, U.S. Market, Leptokurtic Feature, Jump-Diffusion Process, Kou Model, Nonlinear Least Squares.**

I. Introduction

Theoretical models that are interested in evaluating options are generally based on two key elements: the process of the underlying asset and the market price of the risk factor. The Black-Scholes model is based on the assumption of a lognormal diffusion process with a constant instantaneous volatility. Being the benchmark for derivative assets valuation, this model has been, during the last thirty years, the target of several empirical studies that have revealed a number of limitations. On the one hand, the assumption of log normality of the underlying asset has been widely rejected by the ARCH literature. On the other hand, the assumption of a diffusion process was also rejected by the existence of heavy tails of the distribution of returns. Finally, the effect of debt raised by Black (1976) and the existence of a possible correlation between the process and the volatility of the underlying asset, Heston (1993), Nandi (2000), indicated a complex relationship between asset returns and volatility. These empirical limits pushed theorists to develop alternate models. Research undertaken thereafter considered three approaches:

- The univariate models: These are models that have maintained the no-arbitrage assumption of the Black - Scholes model, but gave up the assumption of Geometric Brownian Motion. Included are the Constant Elacticity Variance model (CEV) of Cox and Ross (1976) and Cox and Rubinstein (1983) and more recently the trinomial or implied binomial tree models of Derman and Kani (1994) and Dupire (1994).
- The stochastic volatility models: These models are based on the assumption of a volatility of the underlying asset evolving in a stochastic manner by following a diffusion process, Heston (1993), Hull and White (1987), Wiggins (1987), and the hybrid jump-diffusion process of Duffie and al. (2000).
- The Jump diffusion models: thathas replacedthe underlying asset classical diffusion process. Out of which The Merton Model (1976) remains the most popular.

Very recent studies have attempted to combine these three approaches, such as studies by Jones (2003) and Skiadopulos (2000) who have respectively proposed a stochastic generalization of the CEV process and the binomial tree model.

This paper proposes to compare the empirical performance of three alternatives to the Black-Scholes model that belong to three different classes. First, the ad-hoc Black-Scholes model that is praised by practitioners for its simplicity and effectiveness. It is simply the Black Scholes classical model with a daily implied volatility calibration from option pricing. Although such procedure seems unorthodox and inconsistent with the assumptions of the Black-Scholes classical model, it provides quite suitable results in the evaluation of options[1]. The second model is the Constant Elasticity of Variance model developed by Cox and Ross (1976) better known by the

[1]In a famous article entitled "How to get the right option price with the wrong model?"Berkowitz (2001) showed that, thanks to the daily volatility calibration, the Ad Hoc BS model arrived to provide close performances to those of stochastic volatility models in terms of evaluation"in sample".

abbreviation CEV model that belongs to the class of univariate models. The third alternative is the jump-diffusion model of Kou (2002) which proposes a hybrid process for the underlying asset, consisting of a first "diffusion" component the same as in the Black Scholes model and a second "jump" component following a double exponential process. Such a model allows us to understand two major empirical phenomena: The Kou model leads to a probability distribution with heavy tails (a frequentlyobserved phenomenon of the underlying assetsdistributions) which simply means a greater probability for extreme values. Then the Kou model is able to integrate the phenomenon of negative skewness (more probability for negative outcomes) through the jump signs[2].

The empirical approach will be structured as follows: We begin by presenting the structure of the database used in this study. Options traded on the Chicago Board Options Exchange during the year 2007 for the SP 500 stock index for a total of 12,499 call options. Thenwe conducta comparative analysis betweenthe ad hoc BS, the CEV model and the jump-diffusion Kou model.This analysisaims to verifythevalidity of the assumptionsmade byeach of these models by comparingthe modelpricesto market options prices.The comparative analysiswill alsodetectany structuralbias thatwould affect theperformanceofeach of the three theoretical models.

II. The Constant Elasticity Variance model of Cox and Ross (1976)

Cox and Ross (1976) developed a pricing model of calls that verifies the negative relationship between return volatility of the underlying asset and its price. In this model, the variance of returns is a deterministic function of the underlying asset price and its elasticity with respect to price is constant. Specifically, the model assumes that the instantaneous rate of return of the underlying asset evolves according to the following process:

$$\frac{dS}{S} = \mu \cdot dt + \delta \cdot S^{\theta - 1} \cdot dz \qquad (1)$$

$\mu$is the drift rate of the underlying asset return,

$\delta.S^{\theta-1}$is theinstantaneousstandard deviation of the underlying asset return withδ a strictlypositive constant,

dz : is astandardWienerprocesswhichfollows a normaldistribution with expectation $E(dz)=0$ and variance $Var(dz)=dt$.

The major difference with the Black-Scholes model is that the volatility of returns of the underlying asset $\delta.S^{\theta-1}$ is based on the price of the asset. However, If $\theta = 1$, the CEV model coincides with the Black –Scholes model. Whereas, when θ deviates from 1, the process that characterizes the underlying asset becomes non-stationary. The negative correlation between asset prices and volatility, as evidenced by several empirical studies will be checked only if $\theta < 1$.

However, this model continues to consider the parameters δ and θ as constants, which does not seem to be a very realistic assumption especially when it comes to evaluating options on stock indexes. Indeed, if one refers to the idea of a constant negative elasticity, we could end up in a vicious circle, since any decline in the stock index will increase volatility. This latter increases market fears and causes a further decline in the index. With such a mechanism, we may end up with a volatility that tends to infinity along with a stock index which tends to zero. Such a situation is unlikely. One solution to this problem would be to recalibrate the CEV model on a periodic basis to update its δ and θ structural parameters[3].

III. The jump-diffusion model of Kou (2002)

The model is quite simple in its logic. The logarithm of theunderlying asset priceis assumed to followa hybrid jump-diffusion process. The first component of the process is similar to that of Black- Scholes geometric Brownian motion. The secondcomponent correspondsto a"Poisson"process jumpswith amplitudesdistributed according to thedouble exponential distribution.The model assumesthat the underlying asset priceevolves according tothe following process:

$$\frac{dS(t)}{S(t-)} = \mu.dt + \sigma.dW(t) + d\left(\sum_{i=1}^{N(t)} (V_i - 1)\right) \qquad (2)$$

W(t) isastandardBrownian motion,

N (t) isa Poisson processwithafrequency λ,

$\{V_i\}$isasequenceof positive random variablesindependentlyandidenticallydistributedsuchthatY= Log (V) follows a distributionwithanasymmetric doubleexponentialdensity function:

$$f_y(y) = p.\eta_1.e^{-\eta_1 \cdot y} 1_{\{y \ge 0\}} + q.\eta_2.e^{-\eta_2 \cdot y}{}_{\{y<0\}} \qquad (3)$$

$\eta_1 > 1$ and $\eta_2 > 0$

Where $p,q \geq 0$, p+q=,1 represent the probabilities of upward and downward jumps. The drift μ and the volatility σ are assumed to be constants and the Brownian motion and jumps are assumed to be one dimensional.

In other words,

$$Log(V) = Y \overset{d}{=} \begin{cases} \xi_+ \text{ with probability (p)} \\ \xi_- \text{ with probability (q)} \end{cases} \qquad (4)$$

Where ξ_+ and ξ_- are two exponential randomvariableswith means$1/\eta1$and$1/\eta2$, respectively, and

[2]By proposing negative jumps for the underlying asset return, the model affects more probability for negative achievements.

[3]Like the ad hoc BS model, we proceed to the daily calculation of the structural parameters of the CEV model from option prices. For the remainder of this article, we will denote model with calibration by CEV.

the notation $\stackrel{d}{=}$ means equalin distribution.Yis the randomvariablerepresenting thejumpsthat may affect theunderlying asset rate of returns. ξ_+ and ξ_- are respectivelythe amplitudes of theupward and downwardjumps.

$E(Y) = (p/\eta_1)-(q/\eta_2)$ istheaverageamplitudeof the jump.

$Var(Y) = p.q.(1/\eta_1 + 1/\eta_2)^2 + [(p/\eta_1^2) + (q/\eta_2^2)]$ is part of thevolatilityof the underlying assetdue tojump risk.

This will provide:

$$Var(S) = \sigma^2 + Var(Y) \qquad (5)$$

There are three interesting properties of the double exponential distribution which are fundamental to the model. First, the distribution has the leptokurtic feature. This feature that governs the jump size distribution is consistent with the empirical distribution that characterizes the underlying asset rate of return. Then, the double exponential distribution has the memory less property. In other words, the current achievements depend, in one way or another, on the past achievements. Finally, this distribution has a psychological and economic justification. Indeed, it has been demonstrated through several empirical studies that markets tend to have an overreaction and under-reaction towards various good or bad news, Fama (1998), Barberis et al. (1998). We can then interpret the jumps as a market response to new external market information. Thus, in the absence of external information, the price of the underlying asset should move according to a Brownian motion. Good or bad news occur according to a Poisson process and the price of the underlying asset changes in response to this news, according to the distribution that governs the size of the jump. This distribution can be used to model the overreaction (through heavier tails) and the under-reaction (through a larger peak). Therefore, the diffusion model with double exponential jumps can be interpreted as an attempt to build a simple model within the traditional framework of random walk and market efficiency that takes into account investor's attitudes towards risk as well.

The European call valuation formula, according to the Kou jump-diffusion model, is given by:

$$\begin{aligned}\psi_c(0) &= S(0)\cdot \mathrm{X}\left(r+\frac{1}{2}\cdot\sigma^2-\lambda\cdot\xi,\sigma,\tilde{\lambda},\tilde{p},\tilde{\eta}_1,\tilde{\eta}_2;\log\left(\frac{K}{S}\right),T\right)\\ &-K\cdot e^{-rT}\cdot \mathrm{X}\left(r-\frac{1}{2}\cdot\sigma^2-\lambda\cdot\xi,\sigma,\lambda,p,\eta_1,\eta_2;\log\left(\frac{K}{S}\right),T\right)\end{aligned} \qquad (6)$$

X: the probability function of the Kou jump-diffusion model

$$\xi=\frac{p\cdot\eta_1}{\eta_1-1}+\frac{q\cdot\eta_2}{\eta_2+1}-1,\ \tilde{p}=\frac{p}{1+\zeta}\cdot\frac{\eta_1}{\eta_1-1} \qquad (7)$$

$$\tilde{\eta}_1=\eta_1-1,\ \tilde{\eta}_2=\eta_2+1,\ \tilde{\lambda}=\lambda\cdot(\xi+1) \qquad (8)$$

The Kou model also presents analytical solutions for the evaluation of American options, look back options, and other exotic options.

IV. Overview of the Database

The final sample used concerns 12,499 call options on the SP500 stock index traded on the Chicago Board Options Exchange (CBOE) during.

TABLE I – PROPERTIES of the FINAL SAMPLE of the SP500 CALLS

Prices reported in the table respectively represent the calls mid-price, the effective spread (defined as the difference between the bid and ask price of the option divided by its average price) and finally the total number of observations for each sample subcategory moneyness / time-to-expiration. The sample period is spread over the whole of 2007 for a total of 12,499 observations. The moneyness equals $(S-K.e^{-r.t})/K.e^{-r.t}$. S means the spot level of the SP 500. K stands for the strike price, (r) for the risk-free interest rate which corresponds to the maturity of the call and (t) the call time-to-expiration. OTM, ATM and ITM calls denote the out-of-the-money, at-the-money and in-the-money options.

	Moneyness (%)	Time-to-expiration (days) 6-30	31-60	61-100	Sub-total
OTM	[-10;-6]	$0.67	$1.1	$2.31	
		0.482	0.408	0.305	
		33	363	599	995
	[-6;-3]	$1.3	$3.72	$7.84	
		0.317	0.194	0.136	
		571	843	521	1,935
ATM	[-3;0]	$5.75	$12.74	$20.11	
		0.149	0.103	0.083	
		1,221	1,089	581	2,891
	[0;3]	$22.94	$29.6	$37.31	
		0.073	0.064	0.053	
		1,147	1,012	549	2,708
ITM	[3;6]	$82.34	$84.58	$89.13	
		0.024	0.024	0.022	
		693	730	502	1,925
	[6;10]	$49.68	$53.88	$59.95	
		0.039	0.037	0.033	
		831	755	459	2,045
Sub-total		4,496	4,792	3,211	12,499

The final sample is obtained by applying five filters. First, all the options with an average price less than 50 cents were removed. Then the options with a spread[4] that represents more than 50 % of the average call price are removed. These first two filters are meant to eliminate calls with a large spread in relation to bid-ask quotations reported by the database. We also removed options with a moneyness which deviates from the range [-10 %, 10 %]. Indeed, the options that are deep out-of -the-money (OTM) or deep-in -the-money (ITM) are illiquid and have a low time value which substantially affects the predictive power of the estimated parameters value. Next, we eliminated options with less than 6 days or over 100 days to

[4]The spread of an option is defined as the difference between the ask price and bid price divided by the mid-price of this option.

expiration. The former have almost zero time premiums while the latter are illiquid. Finally, all options that do not meet the no-arbitrage assumption are eliminated. The majority of observations eliminated correspond to deep ITM calls.

Table I describes the properties of the final sample of SP 500 calls to be used for our empirical study. The sample is dominated by ATM options with 5,599 observations (44.8 % of the final sample) followed by ITM options with 3,970 observations (31.7 % of the sample) and finally OTM options with 2,930 observations (23.5 % of the sample). Referring to the criterion of time to expiration, we realize that the sample is dominated by options of short and medium term maturities with respectively 4,792 (38.3% of the sample) and 4,496 observations (36 % of the sample). The long-term options represent only 25.7% of the final sample with 3,211 observations. The average price of SP 500 calls varies from $ 89.13 (deep ITM options at long term) to $ 0.67 (deep OTM options at short term). The spread ranges from 2.2% of the call mid-price (ITM call at long term) to 48.2% (deep OTM call at short term).

V. Parameter Estimation and Performance Models

In order to have a clearer view of the limits of the Black-Scholes model, we represented the evolution of the implied volatility as a function of moneyness and time-to-maturity for two days arbitrarily chosen in our sample. We then obtained two surfaces of the implied volatility that highlight the dual structural bias plaguing the BS model (Figure 1).

Indeed, referring to these surfaces, we realize that the implied volatility generated from the BS model is not unique in space or constant in time, which is inconsistent with the hypothesis of log normality of the price of the underlying asset on which is based the Black-Scholes model.

The most dramatic change in the volatility is recorded for short term options with a volatility smile where OTM and ITM options show higher volatilities than ATM options**.** Any theoretical model, which presents itself as a serious alternative to the BS model, should provide a significant improvement mainly to short term options. As the time-to-expiration increases, the change in implied volatility becomes more moderate with a decreasing pace, commonly called the sneer where the most ITM options show the highest volatility.

As both phenomena smile and sneer are synonymous with a probability distribution with negative skewness and excess kurtosis, any acceptable alternative model to BS should propose a distribution that integrates these two aspects. Thus, one can moderate the effect of the time-to-expiration and the moneyness as two generating sources of estimation bias.

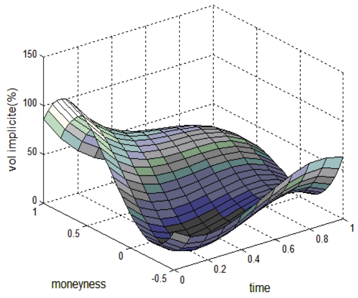

Fig 1.a. SP 500 calls Implied volatility surface, January 2, 2007

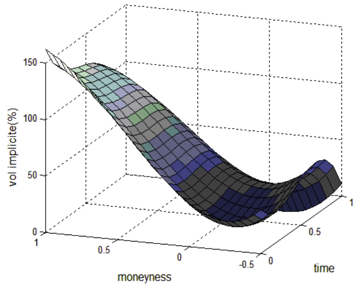

Fig 1.b. SP500 calls implied volatility surface, August 16, 2007

Figure1. The figure shows two surfaces of the implied volatility for two separate days in the sample. The surface traces the evolution of volatility across different levels of moneyness and time-to-expiration. Each point on the surface corresponds to an implied volatility obtained through reversing of the Black-Scholes formula.

A. Alternate models Parameter Estimation

A solution to the parameters estimation problem would be to use the maximum likelihood or generalized method of moments to identify these estimates from the history of the underlying asset. Such a solution can be binding as it requires the collection of a large volume of historical data that eventually leads to low predictive power estimates. In order to address this gap, practitioners and researchers have chosen to derive the estimates of the structural parameters from observed option prices. This solution has introduced the concept of the implied volatility for the BS model. However, the application of such a technique is more complicated with models that involve several structural parameters at the same time and using much more developed mathematical tools than for the case of the BS model.

For this study, we chose to derive the estimates of the structural parameters of the Kou model from instant cross sectional price of options for each day of the sample using the nonlinear least squares. Such a technique can significantly reduce the number of observations required to estimate and leads to a significant improvement in the performance of the evaluation models, Bates (1996 a, b) , Dumas et al. (1998), Bakshi et al . (1997), Melino and Turnbull (1995). The estimation procedure is as follows:

Step 1: For a well-defined sample, we collect m options, such as m is greater than or equal to (n +1) where n is the number of parameters to estimate. In the case of the Kou model, n = 4. C_{i_market} is the market price of the ith call. C_{i_kou} is the theoretical price of the ith call calculated using the Kou model. The difference between these two prices will depend on the vector $\phi = \{\sigma, \lambda, \eta_1, \eta_2\}$. For each option (i), we define:

$$\varepsilon_i(\phi) = C_{i\_market}(t, \tau_i, K_i) - C_{i\_Kou}(t, \tau_i, K_i) \qquad (9)$$

Step 2: We find the vector of parameters that minimizes the sum of squared errors between the observed prices and the theoretical prices of options.

$$SSE \equiv \min_{\phi} \sum_{i=1}^{N} \left|\varepsilon_i(\phi)\right|^2 \qquad (10)$$

These two steps are repeated for each option and for each day in our sample. The objective function SSE is defined as the sum of squared errors, in dollars, of call options prices.

The use of nonlinear least squares should provide a fair comparison between the three models (from as we obtain structural parameters estimated through option prices for both the ad hoc BS model (implied volatility) CEV (θ, δ) and Kou's jump-diffusion ($\sigma, \lambda, \eta_1, \eta_2$).

B. Results of the Estimation

Estimates of the structural parameters of the CEV and the Kou jump- diffusion models are included in Table II.

TABLE II: ESTIMATION of STRUCTURAL PARAMETERS for the CEV and KOU MODELS

The table brings forward the estimates and the corresponding standard deviation for each model parameters. θ and δ are the structural parameters of the CEV model and correspond to the elasticity of volatility and to a positive scalar. σ, λ, η_1, $\eta_2$are the structural parameters to be estimated for the model of Kou. σ designates the portion of the volatility generated by the diffusion process component. λ refers to the average number of jumps per year . η_1 and η_2 respectively control the amplitude of upward jumps (η_1) and downward jumps (η_2). The average amplitude is equal to $p/\eta_1 - q/\eta_2$. p and q denote the probabilities of an upward or a downward jumps. p = q = 0.5.

Parameters	CEV		Kou	
	Mean	Std deviation	Mean	Std deviation
θ	0.8963	(0.0429)		
δ	0.2036	(0.0481)		
σ			0.1024	(0.011)
λ			3.514	(1.174)
η_1			379.179	(180.957)
η_2			13.746	(3.112)

For the CEV model, estimates show a poor negative correlation between the level of the SP 500 index and its volatility. Indeed, as θ tends to 1, the CEV model tends to the Black- Scholes model. Such a result is quite logical since the SP 500 index representing the U.S. equity market has strongly rebounded after the technology bubble and volatility indices have stabilized afterwards. This may explain the poor negative correlation generated by the nonlinear least squares, which only reflect the renewed confidence of market participants.

For the Kou Jump -Diffusion model, estimates are quite reasonable for a fairly diversified stock index, such as the SP 500, especially during stable times. According to the estimation results, the market participants anticipate to achieve an average of 3.514 jumps per year with average amplitude of -3.51 % per jump. The overall average volatility that is measured by the variance represents 1.44 % (being a standard deviation of 11.98%), distributed between 1.05% to the "diffusion" component (or standard deviation 10.24 %) and 0.39 % for the "jumps" component (being a standard deviation of 6.23%) . In other words, the diffusion process contributes to 73% in the overall risk of the underlying asset against only 23% for the "jumps" component.

C. Performance Models

Three criteria were used to conduct a comparative analysis between the three models:

- The mean squared errors: This is the average of the squared differences between the observed price of the option and its theoretical price calculated using each of the three models. This measure gives more weight to in-the-money calls compared to other options in the sample.

- The mean absolute error: At first, the absolute value of the difference is calculated for each option between the observed mid-price of the option and its theoretical price. Then, the average difference was reported at the observed mid-price. This will calculate the percentage of the estimation error for each model. Such a measure tends to give greater weight to evaluation errors related to the calls out-of -the-money at the expense of other options.

- The frequency: This is the number of times where each model has led to the estimation error (mean absolute error) that is lowest compared to the other two models.

This comparative analysis will be conducted by sub-sample "moneyness / time-to-expiration" instead of testing the performance of the three models for the entire sample as a single compact component. Such an approach should allow a better understanding of the elements that may represent sources of estimation bias for our theoretical models.

Tables III, IV and V summarize these criteria divided into 9 sub-samples that are usually based on the moneyness and time-to-expiration. The jump-diffusion model of Kou largely outperforms the CEV and ad hoc Black-Scholes models for all subcategories of the table. The superiority of the model becomes more evident as one moves away from the at-the -money calls and notably for in-the -money ones where the average relative error records its lowest level throughout the sample. This result was predictable since the Kou model was the only one of the three models studied to take into account the aspect of the leptokurtic distribution of the underlying asset. Thus, excess kurtosis can be integrated via the amplitude and frequency of jumps while the negative skewness was present throughout the anticipated jump sign (negative jumps).

We also note that the performance of the model does not seem to suffer a lot from moneyness or time-to-expiration changes. The performance of the Kou model can be explained by several factors. First, the choice of a double exponential distribution, which characterizes the jumps, has improved the quality of estimates since it is more likely to reflect the extreme. Then, the technique of the structural parameters estimation of the model based on the nonlinear least squares has identified estimates based on option prices rather than time series of returns of the SP500 index.

TABLEIII: In-SAMPLE PERFORMANCE EVALUATION MODELS – KOU MODEL

This table shows the three criteria used to assess the quality of the estimate of the jump-diffusion Kou model. These three criteria are, in order of appearance in the table, the mean squared errors, the mean absolute error and the frequency (in parentheses).

Moneyness		Time-to-expiration		
		6-30	31-60	61-100
OTM	[-10;-6]	0.101	0.045	0.09
		46.5%	18.8%	11.04%
		(15)	(322)	(556)
	[-6;-3]	0.058	0.095	0.147
		18.6%	8.87%	3.83%
		(456)	(642)	(431)
ATM	[-3;0]	0.123	0.114	0.188
		6.52%	2.45%	1.72%
		(1102)	(1040)	(493)
	[0;3]	0.169	0.096	0.232
		1.57%	0.82%	0.97%
		(881)	(824)	(380)
ITM	[3;6]	0.191	0.14	0.255
		0.55%	0.49%	0.64%
		(630)	(644)	(401)
	[6;10]	0.311	0.309	0.2
		0.47%	0.5%	0.39%
		(449)	(530)	(464)

Now let's examine the performance of the Constant Elasticity Variance (CEV) model. We note that even though we have applied the same method to estimate the Kou model[5] and have made a daily calibration of the structural parameters, the performance of the CEV model remains widely below those of the Kou model.

The CEV model provides the worst results for in-the - money options where he concedes the second position to the ad hoc BS model for both short-term options, medium and long-term options. We can explain this result by the poor negative correlation between the index level and its volatility especially that we know that 2007 was a relatively "quiet" year[6]. This could be explained by the inability of the CEV model to figure kurtosis excess that has always characterized the performance of indexes.

TABLE IV: In-SAMPLE PERFORMANCE EVALUATION MODELS – CEV MODEL

This table shows the three criteria used to assess the quality of the estimate of the CEV model. These three criteria are, in order of appearance in the table, the mean squared errors, the mean absolute error and the frequency (in parentheses).

Moneyness		Time-to-expiration		
		6-30	31-60	61-100
OTM	[-10;-6]	0.339	0.568	1.22
		88.1%	70.1%	55.2%
		(0)	(4)	(31)
	[-6;-3]	0.332	0.646	1.236
		46.2%	24.5%	14.9%
		(66)	(189)	(88)
ATM	[-3;0]	2.731	3.668	3.297
		26.3%	14.1%	7.6%
		(108)	(42)	(50)
	[0;3]	2.231	2.032	1.862
		6.6%	4.6%	3.1%
		(148)	(68)	(78)
ITM	[3;6]	2.669	8.489	16.318
		2.8%	5.1%	6.4%
		(94)	(4)	(0)
	[6;10]	1.167	5.116	19.862
		1%	2.4%	4.7%
		(118)	(75)	(4)

TABLE V: In-SAMPLE PERFORMANCE EVALUATION MODELS– AD HOC BS MODEL

This table shows the three criteria used to assess the quality of the estimate of the ad hoc BS model. These three criteria are, in order of appearance in the table, the mean squared errors, the mean absolute error and the frequency (in parentheses).

Moneyness		Time-to-expiration		
		6-30	31-60	61-100
OTM	[-10;-6]	0.451	4.654	14.189
		71.3%	146.8%	142.3%
		(18)	(37)	(12)
	[-6;-3]	3.856	12.731	26.815
		129.8%	103.4%	55.4%
		(49)	(12)	(2)
ATM	[-3;0]	8.254	18.894	26.268
		62.7%	30.5%	17.6%
		(11)	(7)	(38)
	[0;3]	5.09	13.89	24.173
		8.3%	7.9%	7.1%
		(118)	(106)	(91)
ITM	[3;6]	1.573	5.785	14.107
		1.9%	2.8%	3.9%
		(107)	(107)	(58)
	[6;10]	0.921	2.884	7.501
		0.9%	1.5%	2.3%
		(113)	(125)	(34)

Finally, the ad hoc Black-Scholes model has usually a good performance for at-the-money short term options. For in-the-money options it provides the best performance

[5]The estimate of the model structural parametersis made with nonlinear least squares technique using cross sectional SP 500 options prices.

[6]The inverse relationship between the price of the underlying asset and its volatility was first introduced to the options with the argument of the leverage effect. In the absence of such an effect for indexes, the only plausible argument to explain this inverse relationship would be the panic effect in the presence of downside market movements.

ahead of the CEV model and rivaling with the Kou model which is another less expected result. Once more, this result confirms that the short term in-the -money options are less sensitive to the choice of models and structural parameters.

However, as one moves towards out-of -the-money options, the generated results are deteriorating in a spectacular way including a time-to-expiration above 30 days. Unfortunately, the daily calibration of volatilities was not able to offset these biases due to the unrealistic assumption of the lognormal asset price. Such a hypothesis is unable to integrate the phenomena that are empirically proved of high kurtosis and non-zero skewness.

D. Estimation errors and regression

We conduct a regression analysis to identify the factors responsible of the estimation errors for all the three models. By estimation error, we mean the mean absolute error $\vartheta_i(t)$ which is a function of $moneyness_i(t)$ the degree of moneyness, $\tau_i(t)$ the time to expiration, and of $spread_i(t)$the spread relative to the i^{th} call observed at date (t). This regression performed using the technique of ordinary least squares will cover all 12,499 SP500 calls. The regression equation is of the following form:

$$\vartheta_i(t) = \beta_0 + \beta_1 \cdot moneyness_i(t) + \beta_2 \cdot \tau_i(t) + \beta_3 \cdot spread_i(t) + \varepsilon_i(t) \quad (11)$$

Regression for SP500 options shows that, regardless of valuation models, all variables have a significant explanatory power at the confidence level of 1% estimation errors. In other words, the estimation errors of the three valuation models are, in part, due to moneyness, time-to-expiration or spread bias.

The magnitude of this bias differs, however, from one model to the other. The moneyness bias is the highest for the ad hoc Black-Scholes model. The percentage of the estimation error for this model is expected to increase by 5.347 points every time the moneyness decreases by one point. Yet, the bias of the moneyness decreases with the CEV model. Hence, the error estimation should increase by 1.517 points every time the moneyness decreases by one point. That is to say that the CEV model offers a better diffusion process than the ad hoc Black-Scholes model. Finally, the bias of moneyness is lower for the Kou jump-diffusion model. The error estimation of this model should increase by only 0.319 points every time the moneyness decreases by one point. This improvement is due to a better process modeling of the underlying asset, thus a better volatility estimate thanks to the introduction of jumps in addition to the diffusion process.

The bias of the residual time is much more discreet than the moneyness for all of the three models. This is due to the daily calibration which allows updating the structural parameters. Such a result is consistent with other studies that have shown that with such a calibration, the Black-Scholes model was able to mimic, in an acceptable manner, the stochastic volatility models, Bakshi et al. (1997), Bates (2003), Berkowitz (2001).

TABLE VI: REGRESSION RESULTS

The table shows the regression results for(11). The regression is performed for each of the 3 models using all 12,499 SP500 calls which constitute our sample. The sample period spans 2007. The coefficient estimates appear in the first line for all 3 models. Figures in parentheses are standard deviations of the estimates. * to mean that the estimate is significant to the 1% error. ** to indicate that it is significant to the 5% error .**

	Parameters				
Models	Constant	Moneyness	Time	spread	R^2
Ad hoc BS	0.233***	-5.347***	-0.207**	1.879***	0.306
	(0.023)	(0.272)	(0.098)	(0.155)	
CEV	0.085***	-1.517***	-0.113***	0.848***	0.604
	(0.004)	(0.055)	(0.019)	(0.027)	
KOU	0.023***	-0.319***	-0.128***	0.325***	0.438
	(0.002)	(0.025)	(0.009)	(0.015)	

The same observation is valid for the CEV model that has been able best to mitigate the time-to-expiration bias factor. This performance is due to two factors. First, the daily calibration of the model parameters has been updated daily. Then, using the process proposed by the CEV model, the volatility does not change in a purely stochastic manner but is inversely related to the price of the underlying asset, as demonstrated by several empirical studies, Heston and Nandi (2000), Jones (2003), Nandi (1997).

VI. Conclusion

The present work was interested in empirically validating three evaluation options models, the ad hoc Black- Scholes model, the Cox CEV model and the Kou jump-diffusion model using call options, negotiated during the year 2007, on the SP500 index. The Constant Elasticity of Variance model uses a diffusion process with the volatility which is a deterministic and inverse function of the underlying asset price. The Kou model offers meanwhile a hybrid model with a hybrid jump-diffusion process where volatility evolves in a stochastic manner.

In order to perform these calculations, we must first estimate the structural parameters for all of the three models. To do so, we choose the nonlinear least squares econometric technique on cross sectional option prices.

A comparative analysis between the three models, based on the evaluation of the theoretical price of 12,499 options on the SP500 shows a clear superiority of the Kou jump-diffusion model which vastly outperforms the two other models for the entire sample. This result shows that the implied distribution over the underlying asset is generally different from the objective distribution. The first is determined by the mood of market participants and their expectations for the future, while the second is simply based on the history of the underlying asset price without considering the psychological aspect of market participants.

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