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Integrating incomplete fuzzy group preferences in a goal programming model for solving a Value-at-Risk efficient portfolio selection problem

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Abstract*:* **In this paper, we develop a novel goal programming model for solving a Value-at-Risk efficient portfolio selection problem. In this paper, the decision making processes in the financial problem take place in group settings where the most important decisions are made by groups of managers or experts whose preferences are often vague and cannot be estimated in exact numerical values. This paper proposes a new method which allows different decision making group members to express their incomplete fuzzy preferences.**

***Keywords*- goal programming; Value-at Risk; Group decision making; Risk Management; Portfolio selection, Incomplete Fuzzy preferences.**

I. INTRODUCTION

The seminal work by Markowitz [1] opened the era of modern finance, and the mean-variance framework is the root of modern investment theory. However, explicit return and risk cannot capture all relevant information for an investment decision. Therefore, criteria for portfolio selection problems, in addition to the standard expected return and variance, have become more popular in recent years [2]. Goal Programming (GP) in operations research is an appropriate method for solving these problems.

This method seeks a compromise solution considering multiple conflicting objectives. Jones and al. [3] discussed the literature on multi-Criteria decision in portfolio selection context as well as the importance of GP applications to portfolio problem and they presented the available research papers on GP for portfolio selection. The mainly existing goal programming approaches to portfolio selection minimizes a measure of risk, where the measure is the variance return of the portfolio [4, 5] or the Beta coefficient for computational simplicity [6]. However, the use of this frame- work with assets that present returns defined by non-elliptic and non normality distributions may be undesirable since it penalizes equally, regardless of downside risk or upside potential. To deal with these limits, this paper presents a goal programming model based on VaR where the Value-At-Risk (VaR) is used among the decision criteria to describe probabilistically the market risk of a trading portfolio.

Usually, in the stock market, the investors is neither able to establish precisely the exact values associated with the weight of each objective in such multi-objective financial problem. In fact, in the case of weighted GP, determination of the weights remains a difficult problem. Generally it depends on several factors such as the preference structure of the decision maker, the decision space, correlations between the objectives and the multiplicity of actors involved in the decision-making process. Up to now, some related theory studies with the decision maker preference have been given but the parameters preferences are usually subjectively fixed with a single decision maker (DM) [5, 6]. However, ordinarily, financial market consist of individual, establishment, instrument, mediator, expert agent and procedures, which collected borrows and savers in one place. In, some practical situations, due to either the vague nature of human judgment, high order of the preference relation presented by multiple decision makers, the DMs may obtain some preference relations with entries being fuzzy. Also, and sometimes, because of time pressure, lack of knowledge, and the DM's limited expertise related with the financial problem, the DMs may develop an incomplete fuzzy preference relation in which some of the elements cannot be provided.

For this purpose, we propose a goal programming model for solving a multi-person multi-criteria decision making portfolio selection problem where the imprecise importance relations among the goals are modeled using incomplete fuzzy preference relations. The obtained fuzzy goal programming model will be applied to establish a new portfolio selection model that considers the tradeoffs between expected return, Value-at-risk, liquidity and the flexibility of incorporating investor's preferences.

II. MODEL FORMULATION

We consider a one-period model with n assets. The manager is not allowed to hold short positions.

Formally, the asset allocation problem is given by:

$$\max_x \sum_{i=1}^{n} E(R_i)x_i$$

Subject to: (1)

$$P\left(\sum_{i=1}^{n} R_i x_i < R^{low}\right) \leq \alpha$$

$$\sum_{i=1}^{n} x_i = 1$$

$$x_i \geq 0$$

Where x_i and R_i denote the fraction invested in asset i and the return on asset i, respectively. $E(R_p) = \sum_{i=1}^{n} E(R_i)x_i$ is the expected return of the portfolio, where E(.) is the expectation operator with respect to the probability distribution P of the asset returns.

$P(R_p \leq R^{low}) \leq \alpha$ in program (1) is the constraint on shortfall probability that the portfolio's return, $R_p = \sum_{i=1}^{n} R_i x_i$ will not fall below the shortfall return R^{low}.

Lucas and Klaassen [7] interpreted the probabilistic constraint in program (1) using the more popular concept of Value-at-Risk. In effect, the constraint fixes the permitted VaR for feasible asset allocation strategies. Value-at-Risk is the maximum amount that can be lost with a certain confidence level. In the setting of probabilistic constraint in program (1) with $R^{low} \leq 0$, the VaR per dollar invested is $-R^{low}$ with a confidence level of $(1-\alpha)$.

However, the model above has some problems. In fact, this model did not comprise the flexibility of investors' preferences for risk and return. Also, the deterministic model cannot be applied to provide an efficient tool to describe real-financial problems where uncertainty of information, multiplicity of objectives, the vague nature of human judgment and the high order of the preference relation presented by multiple decision makers co-occur. Therefore we develop a Value-at-Risk efficient portfolio selection using Goal Programming model based on incomplete fuzzy preference relation in a group decision making context.

In this paper, the proposed GP model considers several objectives simultaneously such as the rate of return, the liquidity and the Value-at-Risk of portfolios.

Thus, a weighted goal programming model can be formulated as:

$$\text{MinZ} = \tilde{w}_1^- \delta_1^- + \tilde{w}_2^+ \delta_2^+ + \tilde{w}_3^+ \delta_3^+$$

subject to: (2)

$$\sum_{i=1}^{n} E(R_i)x_i - \delta_1^+ + \delta_1^- = R_p$$

$$\text{VaR}_p - \delta_2^+ + \delta_2^- = -R^{low}$$

$$\sum_{i=1}^{n} (\text{PER}_i)x_i - \delta_3^+ + \delta_3^- = (\text{PER})_p$$

$$X \in F\left\{X \in R^n / cX\left(\begin{smallmatrix}\leq\\ \geq\end{smallmatrix}\right) C, X \geq 0, CR\right\}$$

$$\sum_{i=1}^{n} x_i = 1 \quad .$$

$$\delta_j^-, \delta_j^+ \geq 0 \qquad \forall\, j = 12{,}3;$$

$$\tilde{w}_j \geq 0 \qquad \forall\, j = 12{,}3;$$

where VaR_p is the expected Value-at-Risk of the portfolio. R_p and $-R^{low}$ are the goals of the expected return and downside risk respectively. δ^+ and δ^- are the positive and the negative deviation from the target of the goal respectively.

The PER_i is the price-earnings ratio of a stock i, this ratio measures the length of time it takes to cover the price by future income, this ratio is to be minimized.

$\tilde{w}_j$ is the fuzzy weight of the j^{th} goal $\forall\, j = 12{,}3;$

In this paper we propose a GP model to obtain the collective weights vector of several incomplete fuzzy preference relations among the goals.

III. GOAL PROGRAMMING MODEL FOR GENERATING THE WEIGHTS VECTOR OF INCOMPLETE FUZZY PREFERENCE

For convenience, several definitions without proofs are summarized below.

Definition 1

Let $L = (l_{jkd})_{mxm}$ be a preference relation, and then L is called an incomplete fuzzy preference relation, if some of its elements cannot be given by the decision maker d, which we denote by the unknown number π, and the others can be provided by the DM, which satisfy [8]:

$$l_{jkd} + l_{kjd} = 1 \forall j,k \in M, \forall d \in D \tag{3}$$

$$l_{jkd} \in [0,1] \tag{4}$$

$$l_{jjd} = 0.5 \tag{5}$$

Where l_{jjd} Represents the preference degree of the objective j to k, provided by the decision maker d.

Definition 2:

Let $L = (l_{jkd})_{mxm}$ be an incomplete fuzzy preference relation, then R is called an additive consistent incomplete fuzzy preference relation, if all the known elements of L satisfy the additive transitivity

$$l_{jkd} = l_{jzd} - l_{kzd} + 0.5 \quad \forall j,k \in M, \forall d \in D \tag{6}$$

For the convenience of computation, we construct an indication matrix $\Delta = (\varsigma_{jkd})_{mxm}$ of the incomplete fuzzy preference relation $L = (l_{jkd})_{mxm}$, where:

$$\varsigma_{jk\,d} = \begin{cases} 0 & l_{jkd} = \pi \\ 1 & l_{jkd} \neq \pi \end{cases} \quad \forall j,k \in M, \forall d \in D \tag{7}$$

Let $W(w_1, w_2, \ldots, w_n)^T$ be the weights vector of the incomplete fuzzy preference relation $L = (l_{jkd})_{mxm}$, where:

$$\sum_{j=1}^{m} w_j = 1$$

$$w_j \geq 0 \; \forall \; j = 12, .. m;$$

If $L = (l_{jkd})_{mxm}$ is an additive consistent incomplete fuzzy preference relation, then such a preference relation must satisfy [9]:

$$\varsigma_{jkd} \; (l_{jkd}) = \varsigma_{jkd} \left[\frac{m}{2}(w_j - w_k) + 0.5\right] \forall \, j,k \in M, \forall d \in D \tag{8}$$

However, in the general case, Eq. (8) does not hold. Refers to [9], we shall relax Eq.(8) by looking for the weights vector of the incomplete fuzzy preference relation $L = (l_{jkd})_{mxm}$ that approximates Eq. (8) by minimizing the error ε_{jkd}, where :

$$\begin{aligned} Min\, \varepsilon_{jkd} &= (\varsigma_{jk\,d} \; (l_{jkd}) - \varsigma_{jk\,d} \left[\frac{m}{2}(w_j - w_k) + 0.5\right]) \\ &= \varsigma_{jk\,d} \left(l_{jkd} - \frac{m}{2}(w_j - w_k\right) - 0.5 \end{aligned} \tag{9}$$

Thus, we can construct the following multi-objective programming model:

$$Min\, \varepsilon_{jkd} = \varsigma_{jk\,d} \left(l_{jkd} - \frac{m}{2}(w_j - w_k\right) - 0.5$$

Subject to: (10)

$$\sum_{j=1}^{m} w_j = 1$$

$$w_j \geq 0$$

The problem of finding a weights vector can also be formulated as the following programming model:

$$Min \sum_{d=1}^{D} \sum_{j=1}^{m} \sum_{k=1,k\neq j}^{m} = \left(\varsigma_{jk\,d} \left(l_{jkd} - \frac{m}{2}(w_j - w_k\right) - 0.5\right)$$

subject to: (11)

$$\sum_{j=1}^{m} w_j = 1$$

$$w_j \geq 0$$

Solution to the above minimization problem is found by solving the following goal programming model:

$$Min \sum_{d=1}^{D} \sum_{j=1}^{m} \sum_{k=1,k\neq j}^{m} \left(\delta_{jkd}^{+} + \delta_{jkd}^{-}\right)$$

Subject to: (12)

$$\varsigma_{jk\,d} \left(l_{jkd} - \frac{m}{2}(w_j - w_k\right) - 0.5 = \delta_{jkd}^{+} + \delta_{jkd}^{-}$$

$$\sum_{j=1}^{m} w_j = 1$$

$$w_j \geq 0$$

$$\delta_{jkd}^{+} + \delta_{jkd}^{-} \geq 0 \; \forall \; j,k \in M, \forall d \in D$$

Each decision maker d who is a member of the group decision making provides a fuzzy incomplete preferences instead of precise preferences for a pairwise comparison matrix L_d :

$$L_d = \begin{bmatrix} 0.5 & l_{12d} & l_{13d} \\ l_{21d} & 0.5 & l_{23d} \\ l_{31d} & l_{32d} & 0.5 \end{bmatrix} \tag{13}$$

l_{12d} represents the preference degree of objective 1 to 2, the degree is provided by the decision maker d.

IV. VALUE-AT-RISK EFFICIENT PORTFOLIO SELECTION USING GOAL PROGRAMMING MODEL INTEGRATING INCOMPLETE FUZZY GROUP PREFERENCES

By combining the expression of program (2) and that of program (12), we formulate a novel goal programming model for solving a Value-at-Risk efficient portfolio selection problem in a group decision making with incomplete fuzzy preference.

$$Min \sum_{d=1}^{3} \sum_{j=1}^{3} \sum_{k=1,k\neq j}^{3} \left(\delta_{jkd}^{+} + \delta_{jkd}^{-}\right) + w_1^{-}\delta_1^{-} + w_2^{+}\delta_2^{+} + w_3^{+}\delta_3^{+}$$

Subject to : (14)

$$\sum_{i=1}^{n} E(R_i)x_i - \delta_1^{+} + \delta_1^{-} = R_p$$

$$VaR_p - \delta_2^{+} + \delta_2^{-} = -R^{low}$$

$$\sum_{i=1}^{n} (PER_i)x_i - \delta_3^{+} + \delta_3^{-} = (PER)_p$$

$$\varsigma_{jk\,d}\left(l_{jkd} - \frac{3}{2}(w_j - w_k)\right) - 0.5 = \delta_{jkd}^{+} + \delta_{jkd}^{-}$$

$$X \in F\left\{X \in R^n \ /cX \binom{\leq}{\geq} C, X \geq 0, C \in R\right\}$$

$$\sum_{i=1}^{n} x_i = 1$$

$$\sum_{j=1}^{m} w_j = 1$$

$$w_j \geq 0$$

$$\delta_{jkd}^{+} + \delta_{jkd}^{-} \geq 0 \ \forall\, j,k \in M, \forall d \in D$$

The software LINGO package can be used to solve program (14). By solving this model, we can get the weights vector $W(w_1, w_2, \dots, w_n)^T$ of incomplete fuzzy preference relation and we find the proportions x_i to be invested in each type of stock i.

Finally, we note that the approach presented here may be applied for solving a real-life portfolio selection problem in the current complex uncertain multi-objective group decision making context.

V. CONCLUSION

This paper demonstrates how the goal programming approach can be efficiently used to solve the fuzzy multi-objective Value-at -Risk portfolio selection problem in a group decision making context. This paper at hand has two important applied and theoretical contributions. First, it presents a practical optimization goal programming model which introduces a Value-at-Risk criterion for solving a multi-objective portfolio selection problem. Secondly, we propose a goal programming method for deriving the fuzzy weights associated with the goals, where the group decision making provides fuzzy incomplete preferences. Our model presents an ability to consider several conflicting objectives under uncertainty and ambiguity. In the future, we wish to use of the present method in a fuzzy stochastic portfolio selection problem.

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