Journal of Economy & International Finance (EIF)
Vol.3pp.41-75

Empirical estimation of the safety stock using the GARCH model, Historical simulation and extreme value theory: Comparative study

Momtez charfi 1 , Ahmed Ghorbel 2

1 Faculty of Economics and Management of Sfax, University of Sfax, Sfax 3018, Tunisia

2 Faculty of Economics and Management of Sfax, University of Sfax, Sfax 3018, Tunisia

Abstract

Safety stock (SS) is an appropriate tactic for dealing with demand and supply uncertainty in order to avoid stock-outs. In the literature, previous work on SS estimation assumes that forecast error distributions are independent and identically distributed following the normal distribution. These quantiles are related to service cycle levels (CSL), which are important for achieving business objectives. Thus, the aim of this research is to propose two combined empirical methods to determine SS more robustly and compare them with traditional methods according to different supply chain parameters. The first method combined, called Filtered Historical Simulation (FHS), involves combining the GARCH model with the simulation method. The second combination, called Conditional Extreme Value Theory (CEVT), is the combination of the GARCH model with the EVT model. To validate these proposed combined methods, the SS is also estimated using traditional methods, such as simple exponential smoothing (SES), simulation and kernel density estimation (KDE), the GARCH method and the Rolling GARCH method.

The methodology is illustrated using both simulation data and real case data study, with reference to two criteria: Backorders and Tick-Loss (TLE). We used simulated data following an $AR(1)$ process with residuals following the normal and log-normal distribution, then real data representing the actual demands used for the manufacture of corrugated board products by the board manufacturing company.

The results are confirmed using the ANOVA test and show the superiority of methods based on the GARCH model, but its performance varies according to the law of the residuals, the delivery time and the effect of variation in the ϕ parameter.

Keywords: safety stock, CEVT, FHS, TLE, Lead time (LT), Backorders, ANOVA.

ISSN: 2961-6638

1. Introduction

1.1 Research and motivation

Make-to-order manufacturing requires the expectation of an order, a work order or a sales order that can be created for a product before the advanced planning and scheduling process is involved. Forecasting is the basis of planning. Forecasts are not based on demand, but on other parameters, such as lead time, as in [2]. Lead time is the time that flows between the placing of a supplier order and the delivery of the goods to the customer. However, forecasting is seen as the core approach to managing production systems based on requirements planning.

In the literature, there are numerous classifications of types demand [50,46]. However, little attention has been paid to measuring the uncertainty around these forecasts, despite the fact that important applications, such as safety stock (SS) determination and replenishment in many replenishment policies, depend on uncertainty estimation. Research into SS estimation can be divided into two groups.

The first group assumes that the distribution of forecast errors is normal, independent and identically distributed. The second group takes heteroscedasticity into account. In the second group, some research suggests applying SS estimation when demands are extreme.

1.2 Literature review

1.2.1 Normal forecast errors

Traditional forecasting approaches are based on point forecasts, such as exponential smoothing (SES), autoregressive moving average (ARMA), integrated autoregressive moving average (ARIMA). These methods are vital for medium-term applications in particular, and are also known for their simplicity in practice.

However, the extent to which it approximates the demand process has not been sufficiently tested, due to the limited repertoire of alternatives to forecasting models in software. The above-mentioned methods can be used if the demand process is perfectly identified and the forecast errors are therefore independent and identically distributed. These errors are generally assumed to be independent and identically distributed, as in Buffa [14], Fotopoulos et al. [25], Eppen and Martin [21], Potamianos et al. [43], Reichhart et al. [44], Gallego-García et al. [27], and Antic et al. [4]. Itis reasonable to ask whether correct identification in the process is possible given the complex relationships that drive demand. Thus, testing on the aforementioned error assumptions is necessary.

Thesupply forecast does not take account of any deviation from these assumptions, and focuses on comparing different forecast error measures, such as mean absolute percentage error or root mean square error, without analyzing residual autocorrelation and deviations from the statistical distribution [8].According to [51] and [52], inventory control measures such as cycle service level, scaled safety stock and backorders may be underperforming if residuals are autocorrelated and distributed differently.

1.2.2 Non-normal forecast errors

Apart from theoretical models which assume that forecast errors are normal, in practice this is not the case, given the complexity of the supply chain. When the deviation from assumptions is presented as generalized autoregressive conditional heteroscedasticity (GARCH) [11], kernel density estimation (KDE) [47] and historical simulation [37] can be useful for throwing out the size of the uncertainty. In a supply chain case study, Zhang and Kline [57] showed that ignoring time heteroscedasticity can increase inventory costs by up to 30% when demand autocorrelation is very positive.

Syntetos and Boylan [48] examined the empirical performance of forecasting methods by estimating the variability of lead-time forecast errors to improve overall system performance. Trapero et al [52] determined the variability of parametric delay forecast error models by applying GARCH models. Using simulated and real data, they showed that the error standard deviation forecast exhibits temporal autocorrelation for high lead times, and that GARCH models gave relevant results. In addition, the distribution of demand is generally assumed to be known, although in practice future demand has to be forecast.

Consequently, forecast errors are among the various factors required to calculate the SS [32], and determining their precise standard deviation is particularly difficult. Some authors, such as Boute et al [12] and Charnes et al [15], have criticized these assumptions, which can lead to errors in decisions concerning safety stocks and service targets that are not met. Trapero et al [52- 53] used empirical methods based on KDEs and GARCH parametric models to calculate SS.

They have shown that the kernel density estimation technique gives better results for shorter lead times because non-normality dominates. For longer lead times, conditional heteroscedasticity is more important and GARCH models are more suitable [53]. For continuous demand data, non-parametric approaches, such as simulation and KDE, take into account the non-normality of forecast errors.

These techniques can be used when the distribution of forecast errors is not known. The effectiveness of these methods has been demonstrated, for example, in the work of Strijbosch and Heuts [48], Manary et al. [33] and Trapero et al. [52]. Manaire et al [33] attempted to correct for the impact of forecast bias, non-normal forecast errors and forecast errors. The authors found SS reductions of around 15%. Strijbosch and Heuts [48] used an empirical dataset from the pharmaceutical industry to adjust statistical forecasts for demand items. They provide information on the combined performance of forecasting and inventory estimation.

1.2.3 The emergence of extreme requirements

Despite their contributions to research, not all the above-mentioned works have taken into account the presence of irregular data in the supply chain. This is highly relevant to inventory management, where special events and promotions are commonplace. The use of Extreme Value Theory (EVT) improves and adjusts the SS calculation for items that may experience extraordinary demands. In a supply chain case study, this approach has been reported as successful by Gallego et al [26], Avanzi et al [5], Bimpikis and Markakis [10], Biçer [9] and Fałdzinski et al [22]. However, which method should be used if the error forecasts possess both an auto-correlated standard deviation, an unknown density function, and an irregular demand process? To resolve these questions, several combination methods have been proposed in the literature, and in practice, it turns out that combining two or more forecasts gives the best performance compared with a single method [16].

1.3 Objective of the study

According to the literature overview above, previous work on SS estimation assumes that forecast error distrubutions are normal and identically distributed. In order to assess violations of this assumption, there are numerous methods of resolution in the recent literature, such as considering the distribution of forecast errors according to the loi lognormal and using the GARCH model which takes into account the phenomenon of herocedasticity, and using EVT to take the occurrence of extreme requirements.

However, the performance of these methods is not guaranteed, as there is a lack of comparative studies. Consequently, the final objective of this research can be summarized by

ISSN: 2961-6638

the following main research question. How can we improve SS estimation methods to simultaneously consider heteroscedasticity phenomena and the occurrence of extreme demands?

This work proposes to estimate SS using two methods mentioned above, which are also widely applied in finance and insurance to estimate the value at risk, which is defined as the quantile of the distribution studied for a given confidence level. The first method consists in combining the simulation and GARCH methods known as filtered historical simulation (FHS), while the second consists in combining EVT with the GARCH model, known as conditional extreme value theory (CEVT).

The choice of combining EVT and simulation with GARCH is also justified by the success of these methods in providing a sufficient estimate of VaR in the context of risk measurement. For each method, in the first parameters of the GARCH step, we extract the standardized residuals, then, in the second step, we apply either EVT or historical simulation to the residual series obtained from the GARCH model. This two-step procedure differs from traditional combined techniques, such as that applied by Trapero et al [53] to combine kernel estimation and GARCH methods. To our knowledge, this is the first work to evaluate SS using these two methods and to compare their performance with traditional methods, assuming a demand that follows the AR process with non-normal distributions for the residuals. We follow a methodology relatively similar to that of Trapero et al [53], with a few differences, notably in the study case and in the choice of distribution parameters. Trapero et al [53] used optimization to determine the optimal weight of each method used in combination.

In contrast, in our paper, the combination is based on simulation and EVT methods applied to standardized residuals from the GARCH model. In addition, the combination of GARCH models is very useful in estimation methods for other fields, such as stock price volatility estimation [40], value-at-risk estimation [18], crude oil price volatility estimation [56], etc. The novelty of this research concerns the proposal of new combined methods for SS estimation in the case where forecast errors are not normal and also not identically and simultaneously distributed and considering heteroskedasticity phenomena and the occurrence of extreme requirements.

This is the first time that a combination of quantiles has been used to calculate FHS and CEVT. To validate the superiority of the proposed methods, simulation and real data from a case study are applied. In the simulation, two cases are studied:

ISSN: 2961-6638

Case 1: when the forecast error follows a normal distribution,

Case 2: when the forecast error follows a lognormal distribution,

In addition, to validate the statistics and simulation results obtained, the one-way ANOVA technique is applied.

2. Materials and methods

2.1 Estimation of theoretical safety stock

The safety stock can be calculated using different approaches. The most appropriate method depends on the circumstances of an organization. The cost of stock-outs is very difficult to estimate in practice. Safety stock sizing depends on levels of demand uncertainty and corresponding forecast errors, when forecast errors are assumed to be independent and identically distributed. In other words, forecast errors follow a normal distribution with zero mean and constant variance; the safety stock is calculated using equation:

$$SS = k\sigma_L$$

k: is called the safety factor with a chosen service level cycle (CSL).

σ_L : is the standard deviation of total demand forecast errors over a period of L (the reapprovisionnement lead time).

$\phi(.)$ is the standard normal cumulative distribution function. Delivery time (DL) is assumed to be known and constant. In this case, the problem is how to calculate σ_L. A first theoretical approach is considered in which the standard deviation of the forecast error σ_L is estimated on the basis of the demand forecast model. σ_L is estimated using the standard deviation σ_1 which is based on the forecast mean square error (MSE).

$$\sigma_1 = \sqrt{MSE}$$

The cumulative variance forecast error is calculated, then the safety stock is calculated using the well-known theoretical formulas mentioned in equation:

$$SS = k\sigma_1\sqrt{LT}$$

This equation is used when forecast errors are independent and do not vary for longer forecast horizons [6]. However, when demand can be expressed by a single method of the exponential smoothing model (SES) [29]. SS is given:

$$SS = k\sigma_1\sqrt{L}\sqrt{1 + \alpha(L-1) + \frac{1}{6}\alpha^2(L-1)(2L-1)}$$

where: α : is the parameter SES which is constant and varies between 0 and 1.

One of the main drawbacks of the theoretical approach is that it does not take into account the fact that forecast errors are not independent in reality. Furthermore, the true model of each element is not found, and the choice of forecast error model cannot be under the company's control. To overcome this forecast error problem, and the problem when the forecast error distribution is unknown, a second approach is considered in which neither the point forecast model nor its parameters need to be known. This is mainly beneficial in practice, especially when this information is not provided to users.

2.2 Empirical estimation of safety stocks

Various empirical approaches can be used to calculate SS. Some of them assume the normality distribution of forecast errors, but take into account the fact that the variance of the forecast error varies over time, i.e. taking into account heteroscedasticity phenomena. In this case, the SS can be calculated using equation:

$$SS = k.\sigma_L = k.\sqrt{\frac{\sum_{t=1}^{n}(\varepsilon_{L,t} - \varepsilon_L)^2}{n}}$$

where $\varepsilon_{L,t}$ is the forecast error for a certain lead time L, expressed in equation :

$$\varepsilon_{L,t} = y_L - F_L = \sum_{h=1}^{L}(y_{t+h} - F_{t+h})$$

$\bar{\varepsilon}_L$ is the average error during the time delay

y_L and F_L are respectively the actual and predicted values during the delay.

Some other of these approaches take into account the non-normality of forecast errors (the distribution of forecast errors is unknown) due, for example, to the promotion periods and complexity of markets; the safety stock is calculated as the quantile (denoted by Q) of the distribution of forecast errors at the probability defined by the CSL.

2.2.1 Simple exponential smoothing (SES)

According to Gardner [28], SES is the most widely used forecasting method in the short term. It is an easy-to-understand and efficient procedure to use when the underlying The demand model is composed of level and random components. Even when the underlying demand process is more complicated, exponential smoothing can be used as part of an updating procedure.

The SES method is a $t+1$ forecasting method. It is used with trendless time series. The principle of this calculation method is to give more importance to the last observations made. Consequently, SES is used as expressed in equation:

$$MSE_{L,t+1} = \gamma(\varepsilon_{L,t})^2 + (1-\gamma)MSE_{L,t}$$

where: : $\sqrt{MSE_{L,t+1}}$: is the forecast for σ_L at time $t+1$.

$\varepsilon_{L,t}$: the cumulative forecast error during the deadline.

γ: a constant smoothing parameter.

Its value lies between 0 and 1. Following Morgan's proposal [31], the value of γ and the initial value of the equation are optimized by minimizing the mean-squared errors.

2.2.2 Historical simulation (HS)

Historical simulation is used as the most popular and efficient method [40]. The main feature of this method is that it is relatively simple to set up, not making any assumptions about the shape of the distribution and its dependence on data availability and sample size. Although predicting the future involves inherent uncertainty, a point forecast can be very useful for quickly describing the expected general path with less complexity. The first step before sizing the SS is to calculate the point forecast. This value is then used together with the forecast error for quantile forecasting approaches. The SES method is used in this work to find the point forecasts [28], it is formulated as mentioned:

$$F_{t+h} = \alpha' \gamma_t + (1-\alpha')F_t$$

where $0 < \alpha' < 1$, h is the forecast horizon. Note that the lead time demand forecast F_L is expressed as mentioned in equation:

$$F_L = \sum_{h=1}^{L} F_{t+h} = L * F_{t+1}$$

Next, the empirical quantile method is a simple approach to estimating SS. It is based on the empirical distribution of historical data on lead-time forecast errors expressed according to the equation: :

$$SS = Q(CSL) - point\ forecast$$

ISSN: 2961-6638

where Q(CSL) is the quantile of the empirical distribution at CSL

2.2.3 KERNEL Density Estimation (KDE)

This technique represents the probability density function $f\ (x)$ of lead time forecast errors, without the need for assumptions about the data distribution. The KERNEL density formula for a series X at a point x is given by :

$$f(x) = \frac{1}{Nh}\sum_{j=1}^{N} K(\frac{x - X_j}{h})$$

where N: is the sample size,

$K()$: is the KERNEL density method function, it is chosen as the density of a standard Gaussian function (zero expectation and unit variance) ;

h: forecast horizon.

It is shown that the optimal KERNEL function, often called KERNEL Epanechnikov [47], is:

$$K_e(t) = \begin{cases} \frac{3}{4\sqrt{5}}\left(1 - \frac{1}{5}t^2\right) & -\sqrt{5} \leq t \leq \sqrt{5} \\ 0 & \forall t \end{cases}$$

Although the choice of h remains a subject of debate in the statistical literature, the following optimal bandwidth for a Gaussian KERNEL is generally chosen:

$$h_{opt} = 0.9AN^{\frac{-1}{5}}$$

Where A: is an adaptive estimate of the extension given by

A : min (standard deviation, quantile interval /1.34)

The quantile can be estimated nonparametrically using the empirical distribution fitted by the KERNEL approach to lead time forecast errors.

2.2.4 GARCH models

To cope with time variation in the standard deviation of lead time forecast errors, generalized autoregressive conditional heteroscedastic (GARCH) models can be used to capture fat tails and heteroscedasticity [40]. The expression of GARCH models (p, q) is given by the conditional variance of forecast errors at time $t + 1$. This variance is given by :

ISSN: 2961-6638

$$\sigma_{t+1}^2 = \omega + \sum_{i=1}^{p} a_i \varepsilon_t^2 + \sum_{j=1}^{q} \beta_j \sigma_t^2$$

where q is the lagged conditional variance terms (σ_t^2) and p is the lagged squared error terms (ε_t^2).

Note that the SES method in the previous equation is a special case of the GARCH model when $p = q = 1$. Knowing that a GARCH (1,1) is given by :

$$\sigma_{t+1}^2 = \omega + a_1 \varepsilon_t^2 + \beta_1 \sigma_t^2$$

The Ses method is a feature of a GARCH model with $\beta_1 = 1 - a_1$ *and* $\omega = 0$.. We apply the GARCH (1,1) model on the cumulative forecast error of the lead time instead of that of the forecast error, the equation can be rewritten as follows:

$$\sigma_{L,t+1}^2 = \omega' + a_1' \varepsilon_{L,t}^2 + \beta_1' \sigma_{L,t}^2$$

We focus on the GARCH (1,1) model using an overlap approach to estimate $\sigma_{L,t}$ as a new sample becomes available.

Next, the safety stock is calculated using the equation

$$SS = \widehat{D} + \mathrm{Q(CSL)}\ \hat{\sigma}_L - point\ forecast$$

$\widehat{D}$ is the demand forecast for a certain lead time LT,

$\hat{\sigma}_L$is the standard deviation of the forecast error for a certain lead time LT,

Q(CSL) is the quantile of the standard normal distribution at CSL and the point forecast is given in equation.

2.2.5 Extreme value theory (EVT)

Fisher et al [24] specified the shape of the limiting distribution by appropriate normalization of maxima. Two traditional approaches are used in the literature to study extreme events.The first is direct modeling of the distribution of minimum or maximum realizations (block maxima method). The other is to model exceedances above a particular threshold (peak-on-threshold method).

De Haan et al [17] study quantile estimation using value theory. McNeil [34] studies tail estimation for distribution loss severity and estimation of risk measures for demands using EVT. Embrechts et al [20] provide an overview of EVT as a risk management tool. Müller et al. [39] and Pictet et al. [42] study the probability of overruns and compare them with GARCH,McNeil [44]provides a comprehensive overview of EVT for risk management.

ISSN: 2961-6638

McNeil and Frey [36]establish a new two-step approach to estimating tail risk measures for heteroskedastic time series, such a method is a combination of GARCH models and EVT. Some applications of EVT to finance and insurance can be found in Embrechts et al. [19] and Reiss et al. [54].EVT is a powerful framework within which to study the tail behavior of a distribution. Classical univariate extreme value theory was developed by [24]. Let $(X_1, \dots, X_n)$ be independent and the variables identically distributed. Let M_n be the maximum of these variables, $M_n = (X_1, \dots, X_n)$. Under conditions of regularity, we can prove that $M_n$tends towards an asymptotic distribution that depends only on that of the base variable.

Let X be a random variable with distribution F. We denote F_X^u the conditional excess distribution function (CDF) above a high threshold u defined as conditional probability:

$$F_X^u(y) = P(X - u \leq y/X > u) \quad 0 \leq y \leq x_F - u$$

where $y = x - u$ is the excess over u and x_F is the right extremity of F. Following Balkema et al. [56] and Pickands[57], the limiting distribution of F_X^u can be approximated by :

$$G_X^u(x) = \begin{cases} \left[1 - (1 + \frac{\xi}{\sigma}(x-u))^{-1/\xi}\right] & si\ \xi \neq 0 \\ 1 - \exp\left(-(x-u)/\sigma\right) & si\ \xi = 0 \end{cases}$$

whereξ is the tail index and$\sigma > 0$ is the scaling parameter. The F distribution can be expressed in terms of the conditional distribution in excess above the u threshold as follows:

$$F_X(x) = \left(1 - F_X(u)\right) F_X^u(x) + F_X(u)$$

$$F_X^u(x) = \frac{F_X(x) - F_X^u}{1 - F_X(u)}$$

Consequently, the distribution of u overruns is accurately known if the distribution of X is known. The function $F(u)$ can be estimated nonparametrically by :

$$\hat{F}(u) = \frac{1}{n}\sum_{i=1}^{n} I(X_i < u) = 1 - \frac{N_u}{N}$$

where N: is the total number of observations,

N_u: represents the number of exceedances above the threshold u.

After replacing $F_u(y)$ by $G_{\xi,\alpha}^u(y)$, we obtain the following estimate for $F(x)$:

$$\hat{F}(x) = \frac{N_u}{N}\left[1 - \left[1 + \frac{\xi}{\hat{\sigma}}(x-u)\right]^{-\xi}\right] + \left[1 - \frac{N_u}{N}\right] = 1 - \frac{N_u}{N}\left[1 + \frac{\xi}{\hat{\sigma}}(x-u)\right]^{-\xi}$$

Inverting this expression, we obtain an expression for $SS_{t+1/t}^{CSL}$ (unconditional) quantiles associate with a given probability p:

Journal of Economy & International Finance (EIF)
Vol.3pp.41-75

$$SS_{t+1/t}^{CSL} = u + \frac{\hat{\sigma}}{\hat{\xi}}((\frac{N}{N_u}CSL)^{-\xi} - 1) - point\ forecast$$

2.3 Proposed combined safety stock estimation

Two combinations of methods are studied in this research. The first named Filtered Historical Simulation (FHS) involves combining the GARCH model with the Historical Simulation Method (HS). The second combination named Conditional Extreme Value Theory (CEVT) concerns the GARCH model with Extreme Value Theory (EVT).

2.3.1 Filtered Historical Simulation (FHS)

The power of this method lies in its ability to adapt to the presence of phenomenal heteroscedasticity and the asymmetry of the empirical distribution, unlike the traditional simulation method. The application of this method is given by modeling the data by the most appropriate GARCH specification (parametric), deriving standardized residuals and applying the historical simulation method (non-parametric) to calculate the quantile. Next, the safety stock is obtained using the GARCH model equation, where Q(CSL) is the quantile of standardized residuals obtained from the GARCH model.

2.3.2 Conditional Extreme Value Theory (CEVT)

To take into account the phenomena of heteroscedasticity and the occurrence of extreme demands simultaneously, we can combine the GARCH model and extreme value theory by following the same two-step estimation procedure proposed by McNeil and Frey [36]. This method is known as conditional EVT or GARCH-EVT:

Step 1: Fit a GARCH family model to demand data by quasi-maximum likelihood estimation. We assume that innovations are normally distributed when maximizing the log-likelihood function. Once the parameters have been estimated, we can extract the residuals to check the adequacy of the GARCH modeling.

Step 2: We apply EVT to the residual calculation standardized in step 1 to model the innovation tail. In this work, we set a certain threshold and retain any innovation above it as extreme (POT method). The threshold is determined under the condition that only 10% of

ISSN: 2961-6638

innovations are above it weconsider the quantile of innovation distribution to be of the order of 90%.

Next, we estimate the quantiles of innovations as presented in equation where Q(CSL) is not applied to demand but applied to standardized residual series. Finally, we deduce the safety stock for different levels with Q(CSL) being the quantile of the standardized residuals obtained from the GARCH model.

2.4 Performance metrics of the safety stock estimation method

2.4.1 Inventory performance measures

The different forecasting approaches are evaluated and compared through the different curves described in the results section. In this work, two variables are considered in the curves: scaled safety stock and backorders. Inventory investment is calculated as follows: first the prediction interval for each item is calculated, then the average of the upper bound of the prediction interval per item and via items corresponds to inventory investment. Remainder units are calculated as the difference between actual sales and quantile predictions.

In fact, if such a difference is positive, it corresponds to out-of-stock units. These units are added up for each item, and then the average backorder units for all items are calculated.

2.4.2 Tick-Loss Function (TLE)

To facilitate comparison between different approaches to safety stock calculation, we need to add other performance measures in addition to inventory investment and backordering. According to Trapero et al [53], the solution is given by the Tick-Loss Function (TLE) used in economic performance. The associated function averages the asymmetrical costs of under- and over-forecasting. The best approach is the one with the minimum loss value. We combine these approaches to minimize the Tick-Loss. The expression for TLE is given by :

$$TL_{\alpha}(y_t, F_t) = \begin{cases} \alpha|y_t - F_t| & si\ F_t \leq y_t \\ (1-\alpha)|y_t - F_t| & si\ F_t \geq y_t \end{cases}$$

where y_t is the actual value at time t,

$F_t$is the predicted value at time t, the parameter α varies between 0 and 1 and any α-quantile of the predictive distribution is an optimal point forecast. In this work, the target quantile is given by the CSL.

2.5 Implementation of safety stock estimation methods

In this study, we set the targets at 85%, 90%, 95% and 99%. The data set is divided as follows: The first part of the data presents 30% of the data and is used to establish the point demand forecast, primarily by calculating the exponential smoothing parameter and its initial value. The second part is used for the KDE, GARCH, Rolling GARCH, FHS and CEVT methods. The last part is used for the KDE, GARCH, Rolling GARCH, FHS and CEVT methods. The last part is used to test the quantile predictions of the methods considered.

The Epanechnikov kernel smoothing function is used for the KDE; the bandwidth of the appropriate value that is optimal for normal distribution densities is defined [54]. The parameters of the GARCH (1,1) model are estimated on the basis of the maximum likelihood estimation method. The analysis was carried out using R. The R package "rugarch" is used to estimate the GARCH parameters and to extract the standardized residuals, using the "evir" package to estimate the model applied to the filtered data. We use the "forecasts" package to calculate the exponential smoothing parameter in equal parts and the "kde1d" package to provide an efficient implementation of kernel density estimators.

3. Simulation results

To assess the performance of the proposed combined methods, we carried out a Monte Carlo simulation with 100 repetitions. The duration of each repetition was set at 700 realizations.

3.1 Case ∅When the forecast error follows a normal distribution

We have assumed that demand at time t follows an $AR(1)$ process as mentioned as follows:

$$D_t = \mu + \emptyset D_{t-1} + \varepsilon_t$$

where μ is a positive constant, ∅ is the autoregressive parameter and $\varepsilon_t$follows the normal distribution with zero mean while ∅ was allowed to vary between -0.9 and 0.9.

The value of μ used for the simulation is 150.

The TLE on the selected sample is averaged over 100 repetitions, then quantiles that include the following values namely 85%, 90%, 95% and 99%, versus autoregressive ∅ parameters that vary between -0.90 and 0.90 are performed and shown in figures 1 and 2.

ISSN: 2961-6638

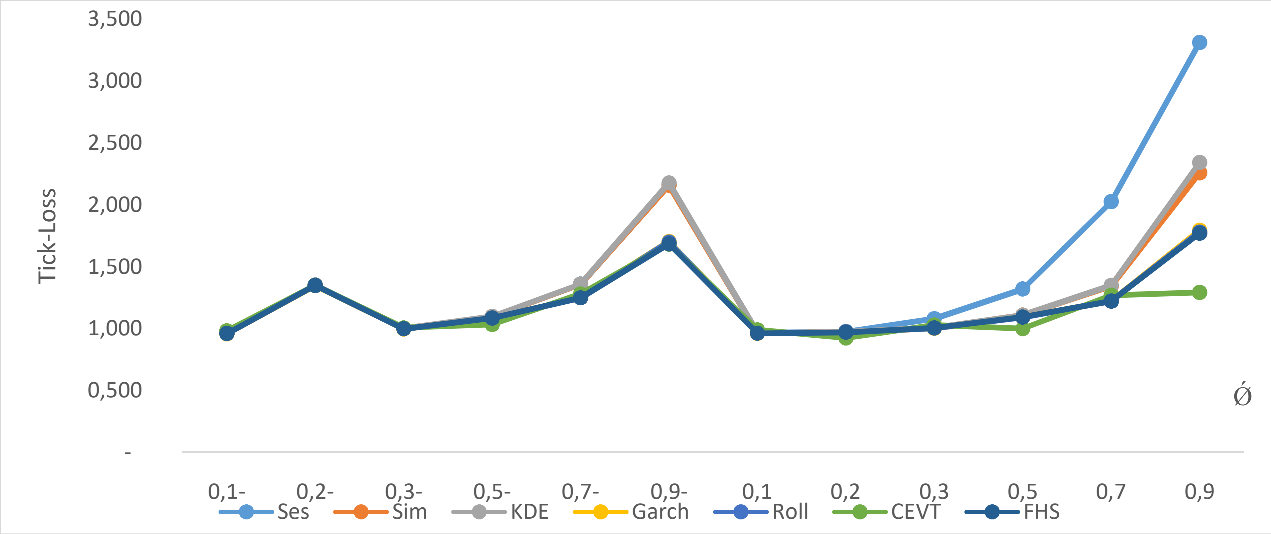


Figure 1: Average tick-loss values for one-week lead times and when forecast errors follow $\boldsymbol{AR(1)_{norm}}$

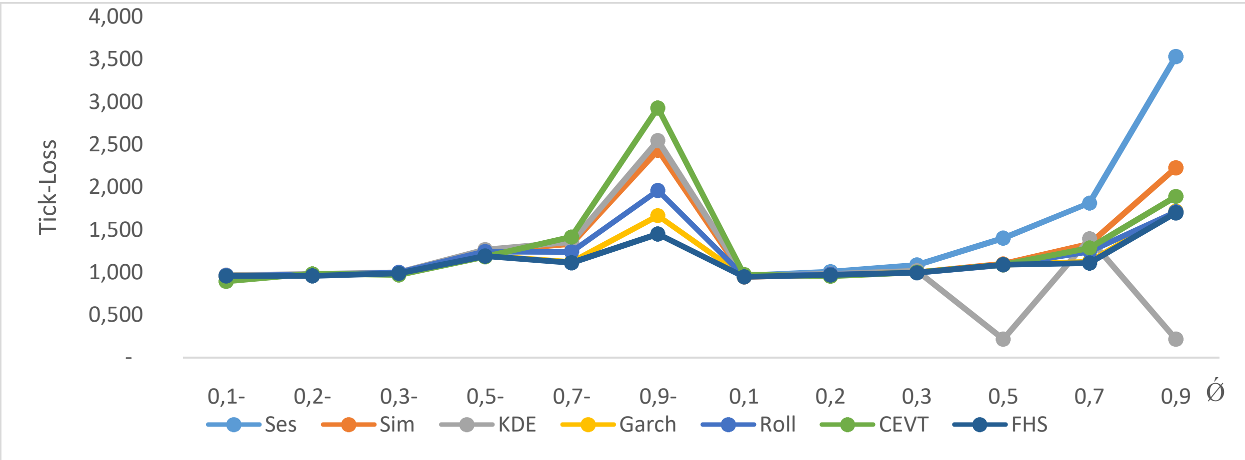


Figure 2: Average tick-loss values for four-week lead times and when forecast errors follow $\boldsymbol{AR(1)_{norm}}$.

Table 1 gives the tick-loss values of each SS measurement method for the four CLS levels and for a one-week and four-week lead time, assuming a demand that follows the AR(1) with normal distribution and $\emptyset = -0.50 \; or \; \emptyset = 0.5$.

For example, the tick-loss value in the case of $\emptyset \; = \; 0.50$ and the one-week lead time is 1.739 at a level of 85% when we apply the CEVT method. This value decreases to 0.205 when CSL is 99%. We choose the four-week period to imply a minimum tick-loss value of 0.191 for the 95% level and 0.08 for a 99% level.

Note the sensitivity of tick-loss to various factors, such as the value of $\emptyset$, the supply lead time and the CSL level. It appears that the CEVT method provides the best performance in the majority of cases, followed by the FHS and GARCH methods. This highlights the fact that there is autocorrelation in the estimation of error variability, which can be explained by the fact that the prediction model does not correspond exactly to the demand generation process.

ISSN: 2961-6638

Traditional methods do not perform as well as GARCH-based methods. The KDE method offers the worst performance in terms of Tick-loss for $\emptyset = -0.5$ and a delay of four weeks.

Table 1. Tick-loss values to $\emptyset = 0.50$ and $\emptyset = -0.50$ when predicting errors following a normal distribution.

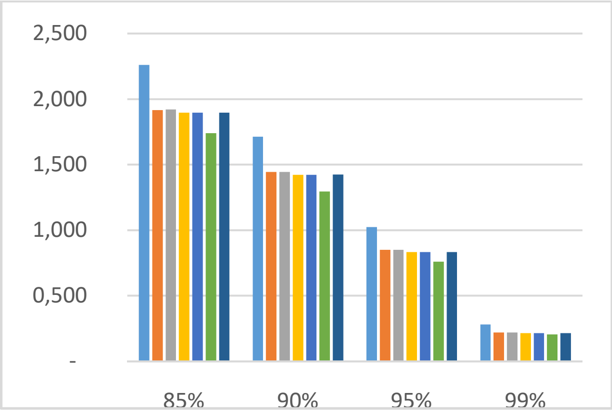
	DL= 1 Week							
	$\phi = 0.5$				$\phi = -0.5$			
CSL	**85%**	**90%**	**95%**	**99%**	**85%**	**90%**	**95%**	**99%**
Ses	2,259	1,713	1,023	0,280	1,896	1,429	0,840	0,219
Sim	1,917	1,445	0,849	0,219	1,895	1,429	0,840	0,221
KDE	1,920	1,445	0,851	0,221	1,897	1,431	0,842	0,222
Garch	1,896	1,423	0,834	0,214	1,880	**1,412**	0,827	0,214
Roll	1,895	1,422	0,833	0,214	**1,879**	1,413	0,827	0,214
CEVT	**1,739**	**1,295**	**0,760**	**0,205**	1,846	1,363	**0,755**	**0,176**
FHS	1,896	1,425	0,834	0,214	1,881	1,413	0,828	0,216
	DL= 4 Week							
	$\phi = 0.5$				$\phi = -0.5$			
CSL	**85%**	**90%**	**95%**	**99%**	**85%**	**90%**	**95%**	**99%**
Ses	2,388	1,818	1,095	0,304	1,916	1,444	1,444	0,223
Sim	1,904	1,433	0,841	0,218	1,907	1,436	1,436	0,222
KDE	**0,311**	**0,266**	**0,191**	**0,080**	1,926	1,454	1,454	0,230
Garch	1,889	1,419	0,830	0,214	1,833	1,364	1,364	0,218
Roll	1,889	1,418	0,830	0,214	1,900	1,428	1,428	0,218
CEVT	1,942	1,422	0,785	0,205	**1,814**	**1,353**	**1,353**	**0,201**
FHS	1,890	1,419	0,830	0,216	1,830	1,362	1,362	0,214

The Tick-Loss values in the table are also plotted in Figure 3. For example, the Tick-Loss value in the case of $\emptyset = -0.50$ and a four-week is 1.814 at a level of 85% when we apply the CEVT method. This value decreases to 0.201 when the CSL becomes 99%.

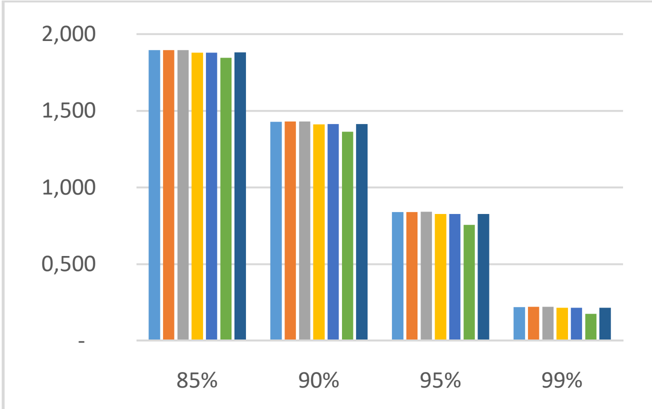
For a one-week, it appears that the FHS method performs best in the majority of cases, followed by the CEVT and GARCH methods. The traditional methods do not perform as well as the GARCH-based methods.

ISSN: 2961-6638

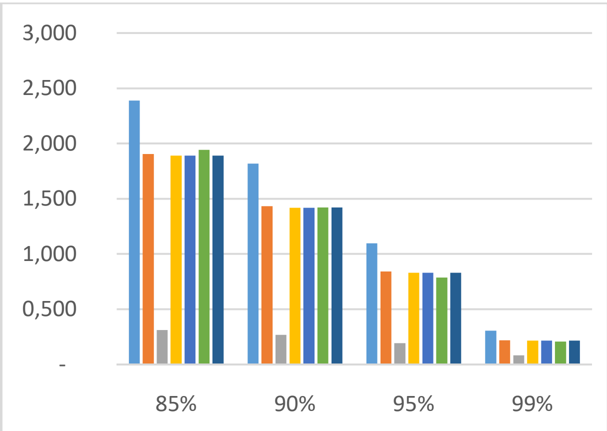
Journal of Economy & International Finance (EIF)
Vol.3pp.41-75



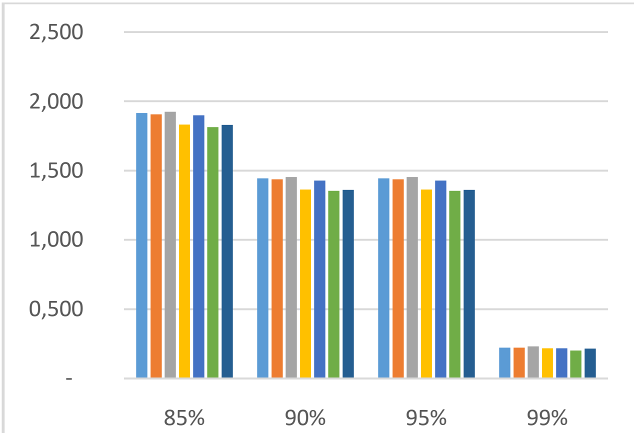
(a) DL= 1 Week Ø = 0,50



(b) DL= 1 Week Ø = -0,50



(c) DL= 4 Week Ø = 0,50



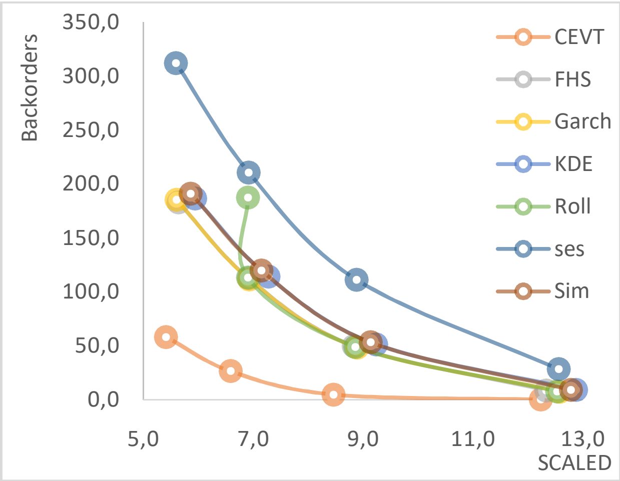
(d) DL= 4 Week Ø = -0,50

Ses Sim KDE Garch Roll CEVT FHS

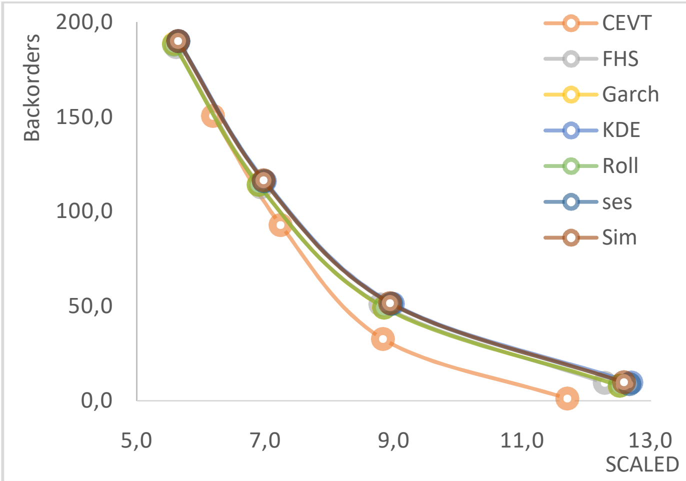
Figure 3. Tick-Loss values in the case of a normal distribution for Ø= 0.50 (a,c) and Ø = -0.50 (b,d) and delays of one week (a,b) and four weeks (c,d)...

Figure 4 shows the trade-off curves in the case of a lognormal distribution for Ø = 0.50 (left) and Ø = -0.50 (right) and lead times of one week (top panel) and four weeks (bottom panel). This figure represents backorders versus inventory investment. Each curve in each panel contains four specific points corresponding to the four CSL targets (85%, 90%, 95% and 99%).

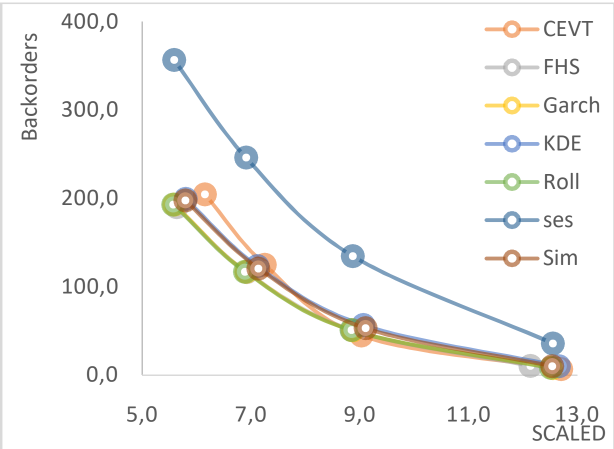
ISSN: 2961-6638



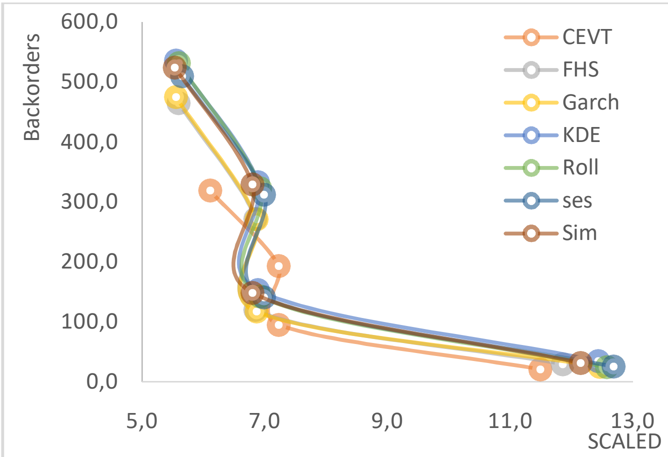
(a) DL= 1 Week ∅ = 0,50



(b) DL= 1 Week ∅ = -0,50



(c) DL= 4 Week ∅ = 0,50



(d) DL= 4 Week ∅ = -0,50

Figure 4: Trade-off curves for AR (1) in the case of a normal distribution for ∅= 0.50 (a,c) and ∅ = -0.50 (b,d) and lead times of one week (a,b) and four weeks (c,d).

We can see in the case of $\emptyset = -0.50$ that the CEVT method performs well in terms of inventory investment and backorders, has targets of 85%, 95% and 99% for a four-week lead time. The 99% level provides a lower level of backorders, but does not give the minimum inventory investment. This makes it difficult to determine the best method. We can also see in the case of $\emptyset = 0.50$ that the CEVT method performs well with the GARCH model, especially for one week.

3.2 Case ∅ where the forecast error follows a lognormal distribution

Let D_t be the demand at time t that follows an AR process (1) as mentioned, where μ is a positive constant, ∅ is the autoregressive parameter and ε_t is independent and identically distributed but is not normally distributed. We added log-normal noise with a mean of 0.9 and a variance of 1.4, while φ can vary between -0.9 and 0.9.

ISSN: 2961-6638

Journal of Economy & International Finance (EIF)
Vol.3pp.41-75

Tick-loss on the retained sample averages over 100 replicates; quantiles comprising the following values, namely 85%, 90%, 95% and 99%, with respect to parameter ∅ are assumed to vary between -0.9 and 0.9 and are shown in figures 5 and 6.

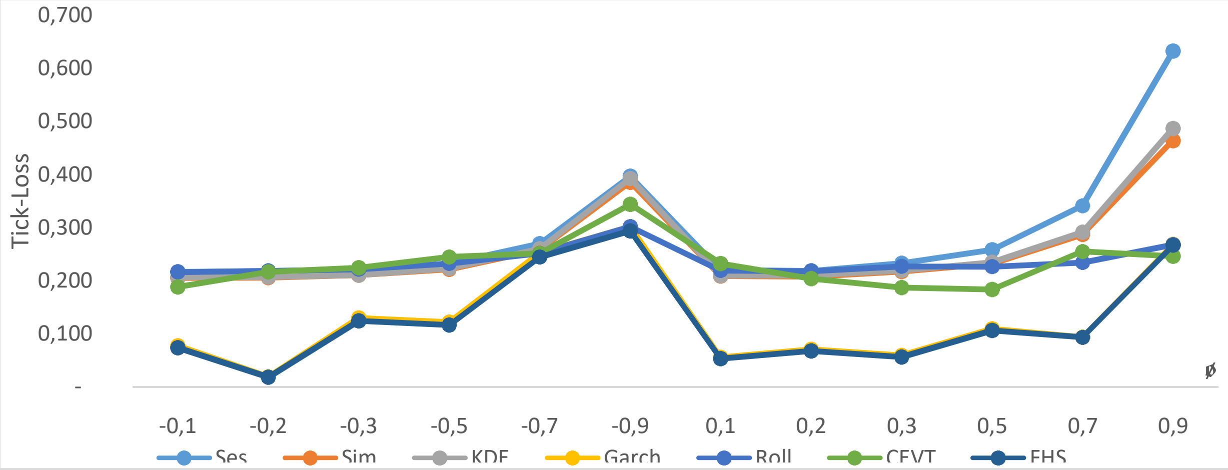


Figure 5. Average Tick-Loss values for one-week lead times and when forecast errors follow a lognormal distribution.

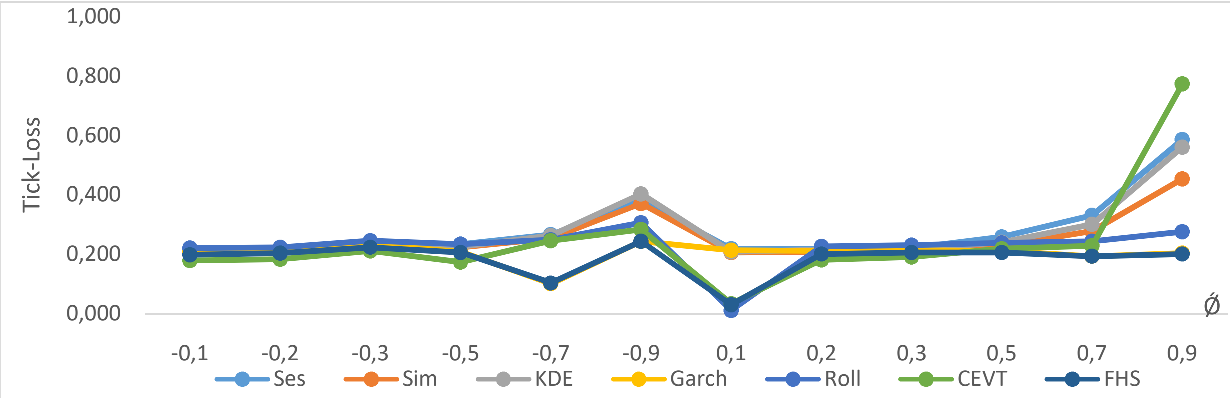


Figure 6. Average Tick-Loss values for lead times of four weeks, when forecast errors follow a lognormal distribution.

Figure 5 includes the case where the delivery lead time (DL) is equal to one, and Figure 6 includes the case where the DL is four weeks. When the DL case is one week, the CEVT method (green line) provided a low value for Tick-Loss for some values of parameter ∅. For other values of ∅ where CEVT is not the best method found in this case, the FHS method (dark blue line) gave better results followed by the GARCH method (yellow line). This proves the better performance of the GARCH model and extreme value theory (CEVT) on the one hand, and the GARCH method and simulation method (FHS) on the other, which is explained by their nature for capturing the deviation from the normality assumption. When the DL case

ISSN: 2961-6638

is four weeks and for negative values of ∅ it seems that CEVT is the best method except when $\emptyset = 0.9$, the FHS gives a minimum Tick-Loss followed by the GARCH method.

For positive values of ∅, we find that the CEVT method is the best for certain values of the autoregressive parameter, otherwise the FHS takes its place, followed by the GARCH method.

The SES method offers the worst performance in terms of Tick-Losses, followed by the KDE and simulation methods.

Figures 5 and 6 show the overall results, with the minimum Tick-Loss values in bold. Nevertheless, the performance of GARCH-based methods can be displayed by choosing particular values of parameter ∅ and plotting the trade-off curves and Tick-Loss value for each quantile of interest. Table 2 gives the Tick-Loss values of each safety stock measurement method for the four CSL levels and for a one-week and four-week DL assuming a demand that follows the AR (1) process with log-normal distribution and $\emptyset = 0.50\ or\ \emptyset = -0.50$. The values in Table 2 indicate the Tick-Loss for a given CSL.

Table 2. Tick-loss values to $\emptyset = 0.50$ and $\emptyset = -0.50$ when predicting errors following a log-normal distribution.

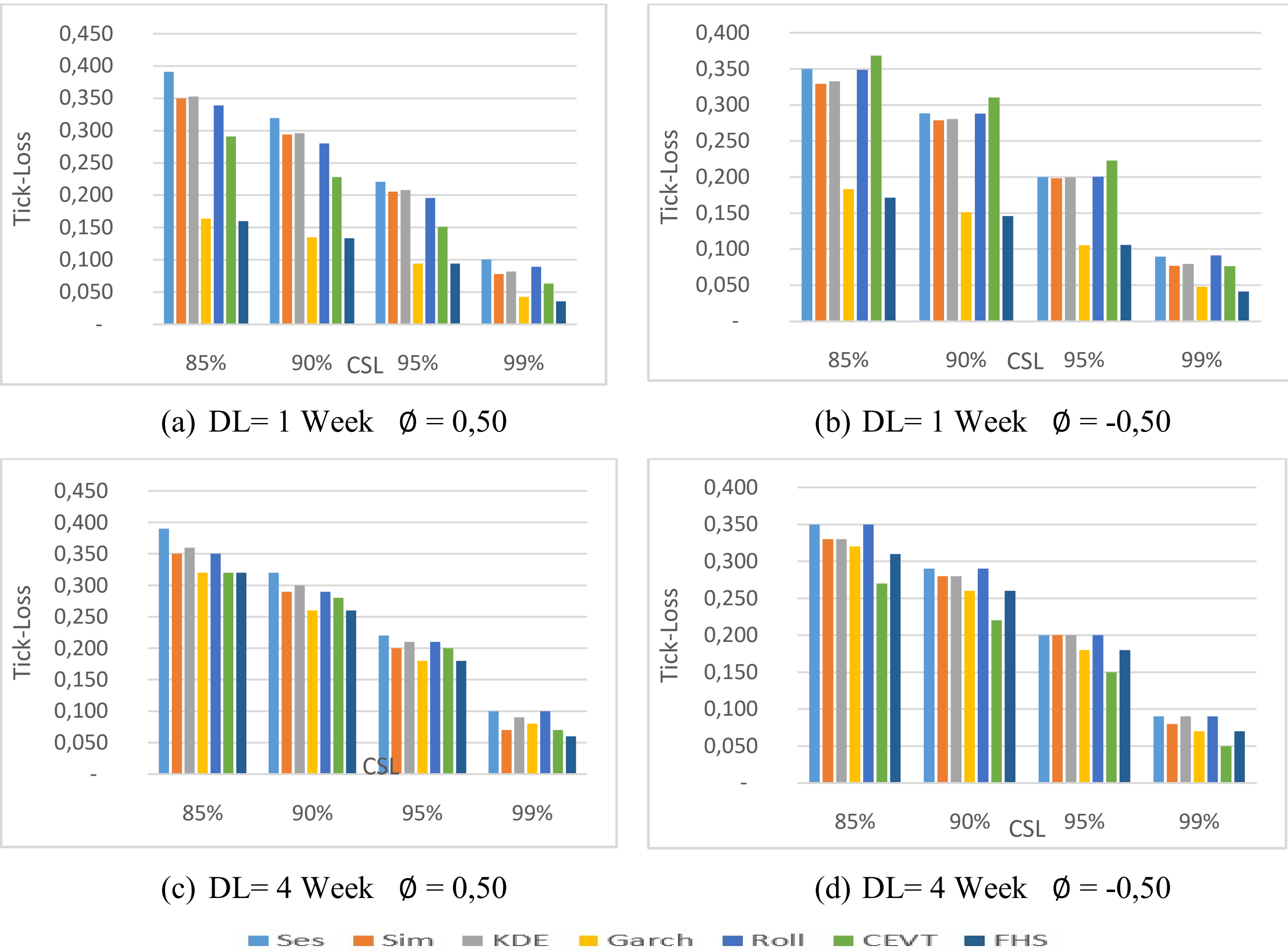
	DL= 1 Week							
	$\phi = 0.5$				$\phi = -0.5$			
CSL	**85%**	**90%**	**95%**	**99%**	**85%**	**90%**	**95%**	**99%**
Ses	0,391	0,320	0,221	0,100	0,350	0,288	0,200	0,090
Sim	0,350	0,294	0,206	0,078	0,329	0,279	0,198	0,077
KDE	0,353	0,296	0,208	0,082	0,333	0,281	0,199	0,080
Garch	**0,164**	0,135	**0,094**	**0,043**	**0,183**	**0,151**	**0,105**	0,048
Roll	0,339	0,280	0,196	0,089	0,349	0,288	0,201	0,092
CEVT	0,291	0,228	0,151	0,063	0,368	0,311	0,223	0,077
FHS	**0,160**	**0,133**	**0,094**	**0,036**	0,171	**0,146**	**0,106**	**0,041**
	DL= 4 Week							
	$\phi = 0.5$				$\phi = -0.5$			
CSL	**85%**	**90%**	**95%**	**99%**	**85%**	**90%**	**95%**	**99%**
Ses	0,390	0,320	0,220	0,100	0,350	0,290	0,200	0,090
Sim	0,350	0,290	0,200	0,070	0,330	0,280	0,200	0,080
KDE	0,360	0,300	0,210	0,090	0,330	0,280	0,200	0,090
Garch	**0,320**	**0,260**	**0,180**	0,080	0,320	0,260	0,180	0,070
Roll	0,350	0,290	0,210	0,100	0,350	0,290	0,200	0,090

ISSN: 2961-6638

CEVT	**0,320**	0,280	0,200	0,070	**0,270**	**0,220**	**0,150**	**0,050**
FHS	**0,320**	**0,260**	**0,180**	**0,060**	0,310	0,260	0,180	0,070

The Tick-Loss values in the table are also plotted in Figure 7. For example, the Tick-Loss value in the case of $\emptyset = -0.50$ and a four-week DL is 0.27 at a level of 85% when we apply the CEVT method. This value decreases to 0.05 when the CSL becomes 99%.

For a one-week DL a minimum tick-loss value of 0.105 is achieved only at the 95% level using the Garch method. In the same case and at the 90% level, the FHS method gives better results. Furthermore, in the case of $\emptyset = 0.50$ and DL of one week, the FHS is more important compared to the other methods given a minimum tick-loss for all CSLs. This result remains unchanged for four weeks. It appears that the FHS method performs best in the majority of cases, followed by the CEVT and GARCH methods. Traditional methods do not perform as well as GARCH-based methods.



(a) DL= 1 Week $\emptyset = 0,50$ (b) DL= 1 Week $\emptyset = -0,50$

(c) DL= 4 Week $\emptyset = 0,50$ (d) DL= 4 Week $\emptyset = -0,50$

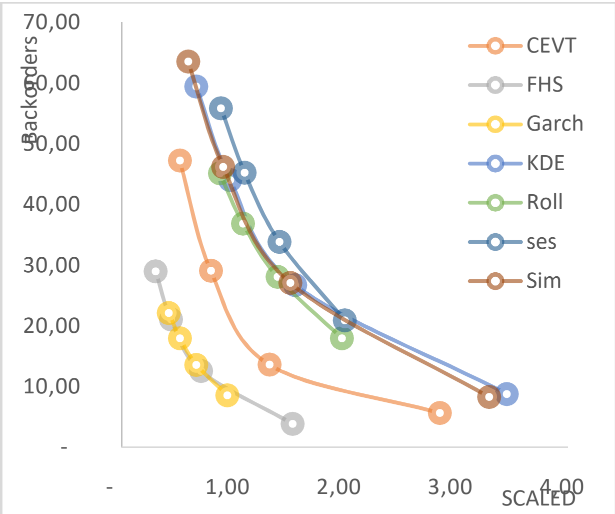
Figure 7. Tick-Loss values in the case of a log-normal distribution for Ø = 0.50 (a,c) and Ø = -0.50 (b,d) and lead times of one week (a,b) and four weeks (c,d).

ISSN: 2961-6638

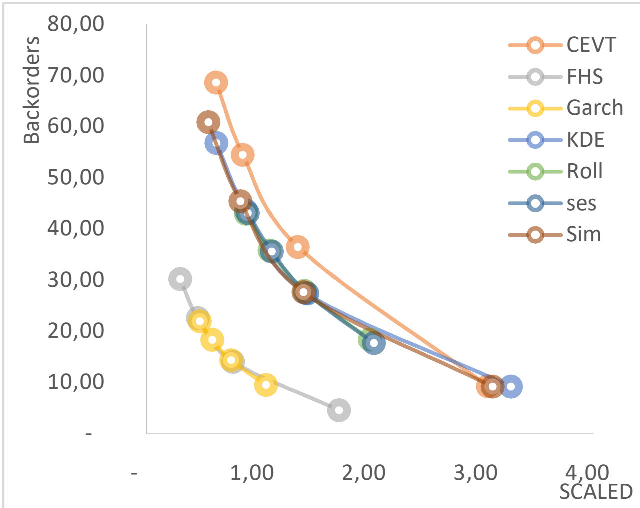
Figure 7 shows Tick-Loss values in the case of a lognormal distribution for $\emptyset = 0.50$ (left) and $\emptyset = -0.50$ (right) and lead times of one week (top panel) and four weeks (bottom panel).

We can notice for all four panels when CEVT gives a higher tick loss value, we find that FHS provides the lowest loss, followed by GARCH and vice versa.

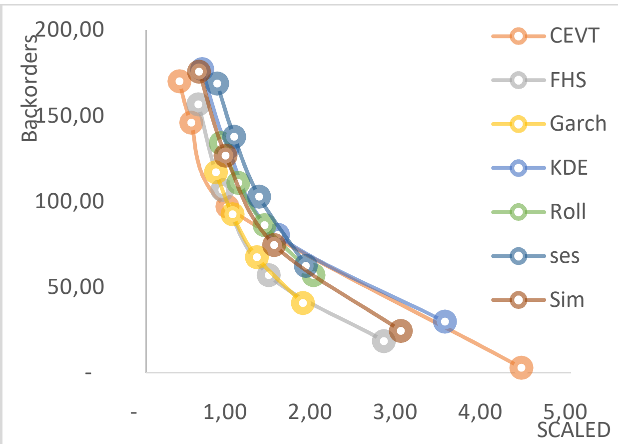
Figure 8 shows the trade-off curves in the case of a lognormal distribution for $\emptyset = 0.70$ (left) and $\emptyset = -0.70$ (right) and lead times of one week (top panel) and four weeks (bottom panel). This figure represents backorders versus inventory investment. Each curve in each panel contains four specific points corresponding to the four CSL targets (85%, 90%, 95% and 99%).



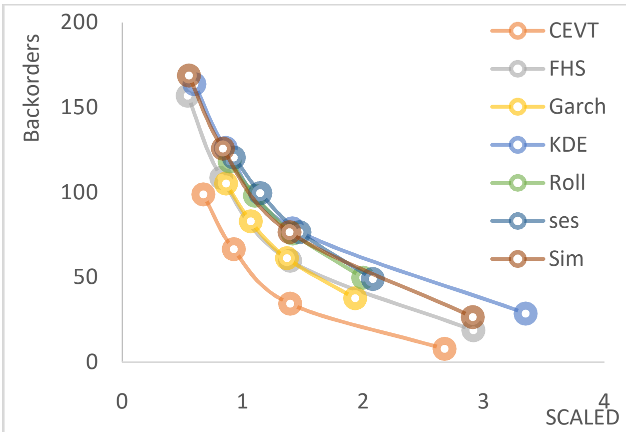
(a) *DL= 1 Week* $\emptyset$ = 0,50



(b) *DL= 1 Week* $\emptyset$ = -0,50



(c) *DL= 4 Week* $\emptyset$ = 0,50



(d) *DL= 4 Week* $\emptyset$ = -0,50

Figure 8. Compromise curves for AR (1) in the case of a lognormal distribution for Ø = 0.50 (a,c) and Ø = -0.50 (b,d) and lead times of one week (a,b) and four weeks (c,d).

It is difficult to determine the best method, since for example in the top left panel and at the 99% level, CEVT provides a lower level of backorders but gives a higher level of investment

ISSN: 2961-6638

in inventory. In contrast, the GARCH method provides a lower level of inventory investment but a higher level of backorders. Despite these conflicts, it's clear that GARCH-based methods and especially GARCH still give good results for all panels and almost all CLS targets.

3.3 ANOVA test

To validate the superiority of the two proposed SS estimation methods, it is necessary to study the significance of the difference between the tick-loss of each SS estimate.

For these reasons, the analysis of variance (ANOVA) technique evaluates the significance of one or more factors by comparing the means of the response variables for the different factors. The null hypothesis states that all population means (factor level means) are equal, while the alternative hypothesis states that at least one of them differs.

An ANOVA was performed with a continuous Tick-Loss response variable at various quantile values taken while accounting for factor diversity. ANOVA require data from approximately normally distributed populations, with equal variances between factor levels.

However, ANOVA procedures work quite well even if the normality assumption is not met, unless one or more laws are highly asymmetrical or the variances are completely different.

Seven methods were chosen Ses, Sim, KDE, GARCH, Roll, CEVT and FHS to calculate the Tick-Loss value at various CSL values taken in the order of 85%, 90%, 95% and 99%.

The one-week lead time was tested with sample sizes of the order of n=200 and n=1000, and for a four-week lead time sample sizes of the order of n =1000 and 1500 were taken. This is tested on two laws (normal law and lognormal law).

Let's suppose we're going to study the effect of the following factors: delivery time, sample size, law and method used on the Tick- Loss value.

The Main Effects Graph function lets you see the influence of one or more category factors on a continuous response.

For example, to evaluate the results of a one-factor controlled ANOVA. We create a main effects plot of the mean sustainability results.

This diagram displays data averages. After fitting a general linear model, you can use factorial plots to create main-effects and interaction plots using fitted means instead of data means.

ISSN: 2961-6638

Figure 9 shows a line connecting the points of each variable. Observing this line indicates whether or not a category variable produces a main effect, while taking into account the overall mean line.

- If the line is horizontal (parallel to the X axis), no main effect is present. The average response is the same for all levels of the factor.
- If the line is not horizontal, there is a main effect. The average response is not the same for all levels of the factor. The steeper the slope of the line, the higher the value of the main effect.

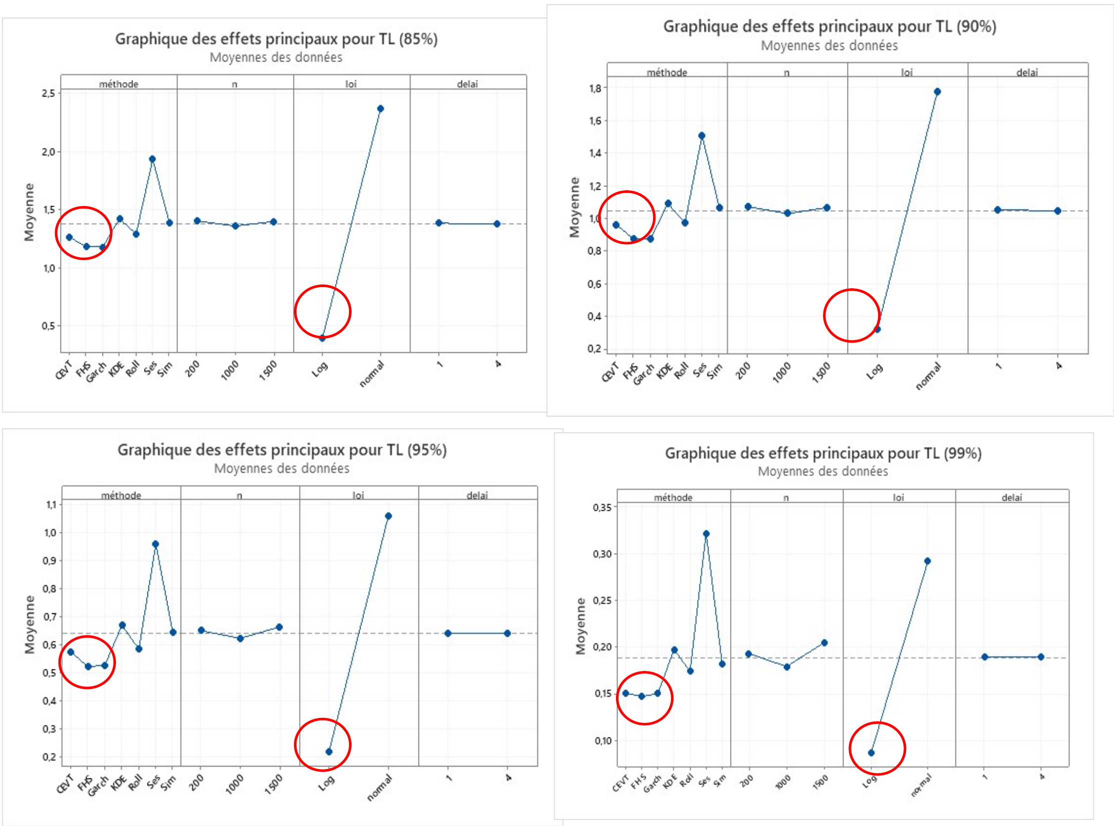


Figure 9: Plot of main effects for TL (85%); TL (90%); TL (95%); TL (99%)

As shown in Figure 9, the lognormal distribution is best suited to giving minimum Tick-Loss values for all quantiles.

The GARCH methods and the two proposed combined FHS and CEVT methods provide minimum Tick-Loss values for all quantiles.

For a sample size of n=1000, Tick-Loss values are minimum. Delay has no influence on Tick-Loss values.

ISSN: 2961-6638

The results of the ANOVA are illustrated in figure 9, which validates the simulation results and confirms that the two combined methods FHS and CEVT give the minimum Tick-Loss values and are better than the classical methods, such as the SES method, the simulation method, the KDE method and the GARCH method. We can also see from Figure 9 that Tick-Loss is best when CSL = 99% and when the delay is equal to one week. The value decreases as CSL increases from 85% to 99% levels. Similarly, as the DL increases, the uncertainty increases, implying an increase in Tick-Loss values. We can also conclude that as the autoregressive parameter increases implying strong autocorrelation between past and actual demands, Tick-Loss values also increase.

3.4 Case study results

3.4.1 Case study dataset

The dataset used in this work comes from a major manufacturer specializing in the production of cardboard packaging and boxes. It specifies the manufacture of customized corrugated packaging products, as well as paper converting and carton manufacturing. The data represents a series of requests with 775 weekly observations for a cardboard product used for product packaging, on which there is the printing and design of this product as well as the name of the company requesting this product. As in the case of simulated data, we used the SES method for point forecasts. The use of this method in an industrial context is highly recommended [48,49]. According to Table 3, real demand is not normally distributed and exhibits a phenomenon of autocorrelation of orders 1 and 2, hence the interest of modeling by an ARMA-GARCH model. Based on Akaike information criterion (AIC), the more appropriate model is AR(1) GARCH(1,1). The estimation parameters for this model are given in Table 3.

Table 3. Descriptive statistics and preliminary tests

Optimal Parameters

	Estimate	Std. Error	t value	Pr(>|t|)
mu	1.0583e+04	5.0609e+02	20.9123	0
ma1	2.5679e-01	2.9854e-02	8.6015	0
omega	1.4105e+05	3.6052e+03	39.1250	0

ISSN: 2961-6638

beta1 9.9853e-01 5.1000e-05 19566.2571 0

Robust Standard Errors:

	Estimate	Std. Error	t value	Pr(>|t|)
mu	1.0583e+04	7.5188e+02	14.0759	0
ma1	2.5679e-01	3.8481e-02	6.6732	0
omega	1.4105e+05	1.4542e+04	9.6998	0
beta1	9.9853e-01	1.1400e-04	8738.1212	0

3.4.2 Case study Estimating safety stocks

Two simulations are carried out with actual data for one and four weeks ahead of time. Figure 10 shows the weekly demand for the item over time. Figure 10 shows the loss values for each method for real demand following the ARMA-GARCH model for one week (right) and for four weeks (left). When the DL case is one week, the CEVT approach reduced the loss to a level of 85%. The same applies when the DL is four weeks. For other CLS, almost all methods give the same value

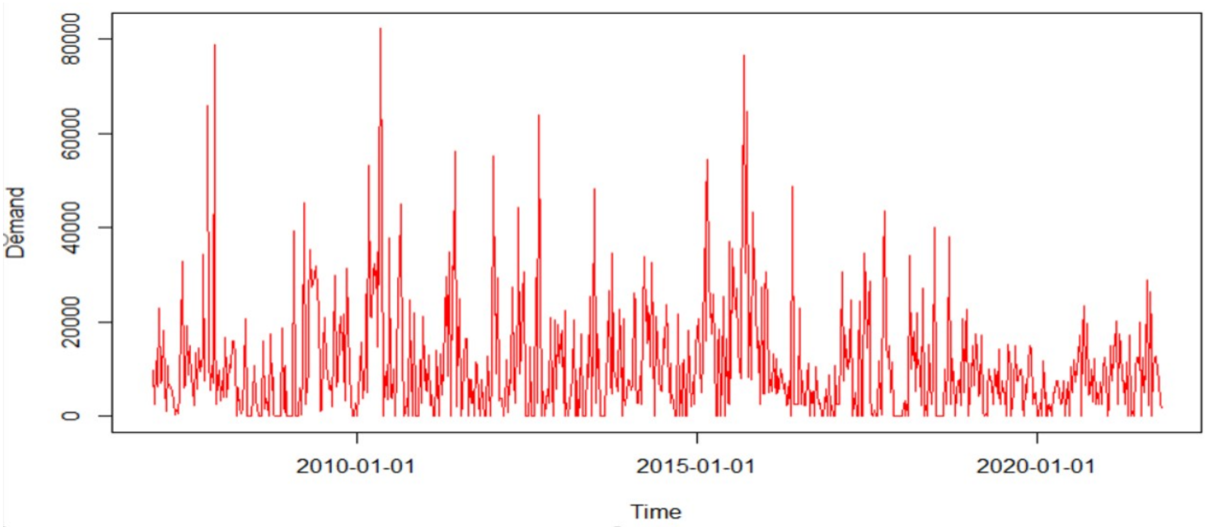
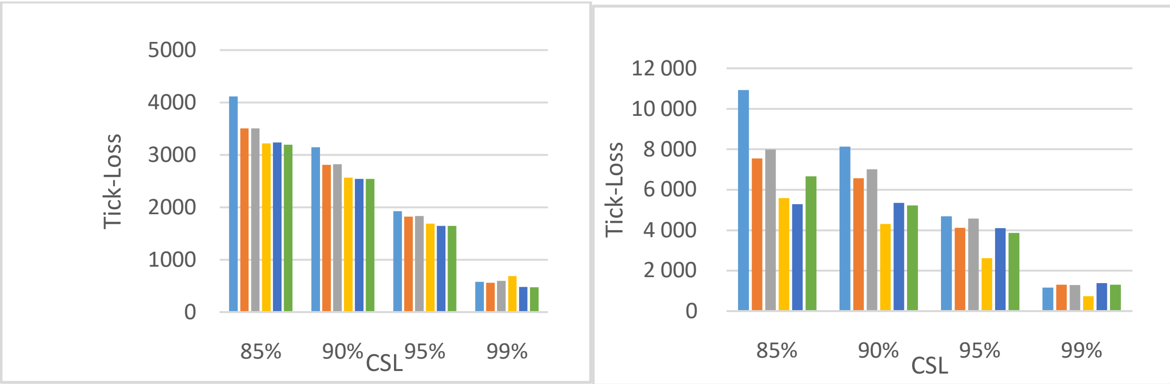


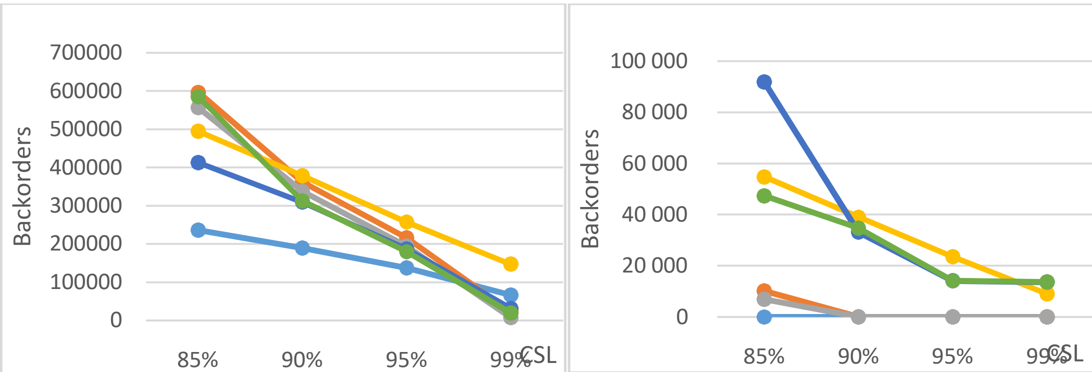
Figure 10: Case study: weekly demands over time



(a) DL =1 (b) DL=4

Figure 11. Tick the loss values of each method for real demand following the ARMA-GARCH model for one week (a) and for four weeks (b).

Figure 11 shows the residuals for each method for the actual demand following the ARMA-GARCH model for one week (left) and four weeks (right).



(a) DL =1 (b) DL=4

Figure 12. Backorders of each method for real demand following the ARMA-GARCH model for one week (a) and for four weeks (b).

In the shorter DL case and at a level of 85% we can see that the GARCH method provides a lower level of investment in inventory but gives higher backorders. In contrast, the CEVT method gives fewer backorders but a higher level of inventory investment. The result of the FHS method is the compromise between these methods.

When the CSL is equal to 90%, the combination of the FHS and CEVT approaches delivers almost the same result. With regard to backorders, both techniques outperform the GARCH method, but in terms of inventory investment, this approach performs better. At the 95% level, the GARCH-based model methods generate the lowest backorders and inventory investment with CEVT improving on FHS and GARCH slightly in terms of backorders. We can see for CSL= 99% that the GARCH approach slightly outperforms the rest of the methods. In the case of longer lead times for CSL = 85%, CEVT performs better in terms of backorders, and there is a slight difference in terms of investment in inventory in favor of FHS followed by GARCH. It should be noted that these three approaches always give better results for 90% and 95% SLC, with a slight difference concerning backorders or inventory investment. The combined FHS technique achieves the best results with the lowest backorders and lowest inventory investment for CSL = 99%.

Generally speaking, such a figure indicates the good performance of GARCH-based methods compared with traditional approaches for each CSL objective for shorter lead times, except for the 99% level where GARCH performs better. For longer lead times, this achievement remains with the exception of the 99% level, where FHS performs best. These results obtained with real data correspond to those obtained with the simulated data performed in the previous section. This coincidence is in the sense that the GARCH method and its combination with simulation and EVT always perform better.

Conclusion

The proposed combination improves on empirical approaches to achieve superior results performance, which is done in a way that is easy to implement with existing forecasting systems, since the only input required is historical data. This work presents the merits of such a combination in a supply chain context for determining safety stocks.

However, further research needs to verify these results in other industrial datasets, for example on slowly evolving items with non-parametric methods. Furthermore, this work is limited to a simulation framework, however, other inventory control policies also need to be investigated.

Another advantage of the proposed approach that is relevant to practice is that it is fully automatic and data-driven, and can therefore be implemented in the context of supply chain forecasting. Empirical techniques can improve the calculation of these SS. In particular, GARCH models are confronted with a time-varying heteroscedastic forecast error. EVT takes into account the occurrence of extreme requirements, and historical simulation need not rely on a fixed distribution. However, if forecast errors are heteroscedastic, do not follow a known distribution and extreme demand is present, then traditional approaches are unsuitable. Two combined empirical methods are proposed to determine SS in a more robust way and compare them to traditional methods published in the literature, under different supply chain parameters. The first method is called FHS, which combines the GARCH model with the

ISSN: 2961-6638

historical simulation method, and the second method is called CEVT, which combines GARCH with extreme value theory. To the best of our knowledge, all previous work uses one of these two approaches combined to calculate SS.

Comparative analyses show the superiority of these combined methods over the Tick-Lossk function for the various CSL targets and for shorter and longer delays. In most cases, CEVT gives the lowest losses; otherwise, FHS takes its place, followed by the GARCH method. These results are confirmed by ANOVA.

Notes The following notations are used in this manuscript:

AIC Akaike Information Criterea

ANOVA Analyze of Variance

AR(1) Auto-Regressive of order 1

ARMA AutoRegressive–Moving-Average

ARIMA AutoRegressive—Integrated-Moving-Average

CEVT Conditional Extreme Value Theory

CSL Cycle Service Level

EVT Extreme Value Theory

FHS Filtered Historical Simulation

GARCH Generalized Auto-Regressive Conditional Heteroskedasticity

GEV Generalized Extreme Value

GPD Generalized Pareto Distribution

LT Lead Time

DL Délai de livraison

iid Independent and Identically Distributed

KDE Kernel Density Estimation

MEF Mean Excess Function

ISSN: 2961-6638

MSE Mean Squared Error

MTO Make-To-Order

MTS Make-To-Stock

SES Simple Exponential Smoothing

SS Safety Stock

TICK-LOSS Tick Loss Function

References

1 Ali, M.M.; Boylan, J.E.; Syntetos, A.A. Forecast errors and inventory performance under forecast information sharing. Int. J. Forecast. 2012, 28, 830–841.

2 Alnahhal, M.; Ahrens, D.; Salah, B. Dynamic Lead-Time Forecasting Using Machine Learning in a Make-to-Order Supply Chain. Appl. Sci. 2021, 11, 10105.

3 Angelidisa, T.; Benosa, A.; Degiannakis, S. The use of GARCH models in VaR estimation. Stat. Methodol. 2004, 1, 105–128.

4 Antic, S.; Djordjevic Milutinovic, L.; Lisec, A. Dynamic Discrete Inventory Control Model with Deterministic and Stochastic Demand in Pharmaceutical Distribution. Appl. Sci. 2022, 12, 1536.

5 Avanzi, B.; Bicer, I.; De Treville, S.; Trigeorgis, L. Real Options at the Interface of Finance and Operations: Exploiting Embedded Supply-chain Real Options to Gain Competitiveness. Eur. J. Financ. 2013, 19, 760–778.

ISSN: 2961-6638

6 Axsäter, S. Inventory Control, 3rd ed.; International Series in Operations Research & Management Science; Springer International Publishing: Berlin/Heidelberg, Germany, 2015; ISBN 9783319157290.

7 Balkema, A.A. et de Haan, L.. Residual Life Time at Great Age. The Annals of Probability, 1974. 2, 792-804, https://doi.org/10.1214/aop/1176996548.

8 Barrow, D.; Kourentzes, N. Distributions of forecasting errors of forecast combinations: Implications for inventory management. Int. J. Prod. Econ. 2016, 177, 24–33.

9 Biçer, I. Dual sourcing under heavy-tailed demand: An extreme value theory approach. Int. J. Prod. Res. 2015, 53, 4979–4992.

10 Bimpikis, K.; Markakis, M.G. Inventory Pooling under Heavy-Tailed Demand; Working Paper; Stanford University: Stanford, CA, USA, 2014.

11 Bollerslev, T. Generalized autoregressive conditional heteroskedasticity. J. Econom. 1986, 31, 307–327.

12 Boute, R.N.; Disney, S.M.; Lambrecht, M.R.; Houdt, B.V. Coordinating lead times and safety stocks under autocorrelated demand. Eur. J. Oper. Res. 2014, 232, 52–63.

13 Bowman AAzzalini, A. Applied Smoothing Techniques for Data Analysis: The Kernel Approach with S-Plus Illustrations. Int. J. Data Envel. Anal. Oper. Res. 1997, 2, 7–15.

14 Buffa, F.P. A model for allocating limited resources when making safety-stock decisions. Decis. Sci. 1977, 8, 415–426.

15 Charnes, J.M.; Marmorstein, H.; Zinn, W. Safety stock determination with serially correlated demand in a periodic-review inventory system. J. Oper. Res. Soc. 1995, 46, 1006–1013.

16 Clements, M.P.; Harvey, D.I. Combining probability forecasts. Int. J. Forecast. 2011, 27, 208–223.

17 De Haan, B. J. Jonas, M. Klepper, O. Krabec, J. Krol, M. S. Olendrzy, K.. An atmosphere-ocean model for integrated assessment of global change. Water, Air, & Soil Pollution, 76(1-2), 283-318, 1994. doi: http://dx.doi.org/10.1007/BF00478343.

18 Echaust, K.; Just, M. Value at Risk Estimation Using the GARCH-EVT Approach with Optimal Tail Selection. Mathematics 2020, 8, 114.

19 Embrechts, P. Klüppelberg, C. Mikosch, T. Modelling Extremal Events for Insurance and Finance. Springer-Verlag, 645 , 1997. ISSN 0172-4568, ISBN 3-540-60931-8.

20 Embrechts, P. Resnick, S. I. et Samoorodnitsky, G.. Extreme value theory as a risk management tool. 1998. North American Actuarial Journal 3(2), 30–41.

21 Eppen, G.D.; Martin, R.K. Determining safety stock in the presence of stochastic lead time and demand. Manag. Sci. 1988, 34, 1380–1390.

22 Fałdzinski, M.; Osinska, M.; Zalewski, W. Extreme Value Theory in Application to Delivery Delays. Entropy 2021, 23, 788.

23 Fildes, R.; Goodwin, P.; Lawrence, M.; Nikolopoulos, K. Effective forecasting and judgmental adjustments: An empirical evaluation and strategies for improvement in supply-chain planning. Int. J. Forecast. 2009, 25, 3–23.

24 Fisher, R.; Tippet, L. Limiting Forms of the Frequency Distribution of the Largest or Smallest Member of a Sample; Cambridge Philosophical Society: Cambridge, UK, 1928; pp. 180–190.

25 Fotopoulos, S.; Wang, M.-C.; Rao, S.S. Safety stock determination with correlated demands and arbitrary lead times. Eur. J. Oper. Res. 1988, 35, 172–181.

26 Gallego, G.; Katircioglu, K.; Ramachandran, B. Inventory Management Under Highly Uncertain Demand. Oper. Res. Lett. 2007, 35, 281–289.

27 Gallego-García, S.; García-García, M. Predictive Sales and Operations Planning Based on a Statistical Treatment of Demand to Increase Efficiency: A Supply Chain Simulation Case Study. Appl. Sci. 2021, 11, 233.

28 Gardner, E.S. Exponential Smoothing: The State of The Art-Part II. Int. J. Forecast. 2006, 22, 637–666.

29 Hyndman, R.J.; Koehler, A.B.; Ord, J.K.; Snyder, R.D. Forecasting with Exponential Smoothing: The State Space Approach; Springer: Berlin, Germany, 2008.

30 Isengildina-Massa, O.; Irwin, S.; Good, D.L.; Massa, L. Empirical confidence intervals for USDA commodity price forecasts. Appl. Econ. 2011, 43, 3789–3803.

31 Jawwad, F.; Palgrave, M. Models at Work: A Practitioner's Guide to Risk Management, Global Financial Market; Springer: Berlin/Heidelberg, Germany, 2014.

ISSN: 2961-6638

32 Lisan, S. Safety stock determination of uncertain demand and mutually dependent variables. Int. J. Bus. Soc. Res. 2018, 8, 1–11.

33 Manary, M.P.; Willems, S.P.; Shihata, A.F. Correcting heterogeneous and biased forecast error at intel for Supply Chain Optimization. Interfaces 2009, 39, 415–427.

34 McNeil, A. 1998. Calculating quartile risk measures for financial return series using extreme value theory. Working Paper, Eidgenössische Technische Hochschule Zürich.

35 McNeil, A. 1999. Extreme value theory for risk managers. Internal Modelling and CADII: Qualifying and Quantifying Risk within a Financial Institution. Risk Waters Group, London,93–113.

36 McNeil, A.; Frey, R. Estimation of tail-related risk measures for heteroscedastic financial time series: An extreme value approach. J. Empir. Financ. 2000, 7, 271–300.

37 Meera, S. The Historical Simulation Method for Value-at-Risk: A Research Based Evaluation of the Industry Favorite 2018. Available online: https://ssrn.com/abstract=2042594 (accessed on 20 March 2021). Appl. Sci. 2022, 12, 10023 24 of 25

38 Morgan, J.P. RiskMetrics. Riskmetrics Technical Document, 4th ed.; Technology Report JPMorgan/Reuters; Morgan Guaranty Trust Company: New York, NY, USA, 1996.

39 Müller, A. et al.. New aspects of electron transfer revealed by the crystal structure of a truncated bovine adrenodoxin, 1998. Adx(4-108). Structure 6(3):269-80.

40 Naik, N.; Mohan, B.R. Stock Price Volatility Estimation Using Regime Switching Technique-Empirical Study on the Indian Stock Market. Mathematics 2021, 9, 1595.

41 Pickands, J. Statistical Inference Using Extreme Order Statistics. 1975. Annals of Statistics, 3, 119-131, http://dx.doi.org/10.1214/aos/1176343003.

42 Pictet, Gabriel, Banza, Baya. Georges, Guiella. et Christine, Ouedraogo. . Rapport d'évaluation : Evaluation de la stratégie de distribution à base communautaire. 1998. Social and behavioral science research (SBSR).

43 Potamianos, J.; Orman, A.; Shahani, A. Modelling for a dynamic inventory-production control system. Eur. J. Oper. Res. 1997, 96, 645–658.

44 Reichhart, A.; Framinan, J.M.; Holweg, M. On the link between inventory and responsiveness in multi-product supply chains. Int. J. Syst. Sci. 2008, 39, 677–688.

ISSN: 2961-6638

45 Reiss, R.-D. and Thomas, M. . Statistical Analysis of Extreme Values: with Applications to Insurance, Finance, Hydrology and Other Fields.1997. 2nd edn, Birkhäuser, Basel.

46 Rožanec, J.M.; Kažiˇc, B.; Škrjanc, M.; Fortuna, B.; Mladeni´c, D. Automotive OEM Demand Forecasting: A Comparative Study of Forecasting Algorithms and Strategies. Appl. Sci. 2021, 11, 6787.

47 Silverman, B.W. Density Estimation for Statistics and Data Analysis; Chapman & Hall: London, UK, 1986.

48 Strijbosch, L.; Heuts, R. Modelling (s, Q) inventory systems: Parametric versus non-parametric approximations for the lead time demand distribution. Eur. J. Oper. Res. 1992, 63, 86–101.

49 Syntetos, A.A.; Boylan, J.E. Demand forecasting adjustments for service-level achievement. IMA J. Manag. Math. 2008, 19, 175–192.

50 Syntetos, A.A.; Boylan, J.E.; Croston, J. On the categorization of demand patterns. J. Oper. Res. Soc. 2005, 56, 495–503.

51 Syntetos, A.A.; Nikolopoulos, K.; Boylan, J.E. Judging the judges through accuracy-implication metrics: The case of inventory forecasting. Int. J. Forecast. 2010, 26, 134–143.

52 Trapero, J.R.; Cardós, M.; Kourentzes, N. Empirical safety stock estimation based on Kernel and GARCH models. Omega 2018, 84, 199–211.

53 Trapero, J.R.; Cardós, M.; Kourentzes, N. Quantile forecast optimal combination to enhance safety stock estimation. Int. J. Forecast. 2019, 35, 239–250.

54 Trapero, J.R.; García, F.P.; Kourentzes, N. Impact of demand nature on the bullwhip effect. Bridging the gap between theoretical and empirical research. In Proceedings of the 7th International Conference on Management Science and Engineering Management: Focused on Electrical and Information Technology; Xu, J., Fry, J.A., Lev, B., Hajiyev, A., Eds.; Springer: Berlin/Heidelberg, Germany, 2014; Volume II, pp. 1127–1137.

55 Trapero, J.R.; Pedregal, D.J.; Fildes, R.; Kourentzes, N. Analysis of judgmental adjustments in the presence of promotions. Int. J. Forecast. 2013, 29, 234–243.

ISSN: 2961-6638

56 Wu, C.; Wang, X.; Luo, S.; Shan, J.; Wang, F. Influencing Factors Analysis of Crude Oil Futures Price Volatility Based on Mixed-Frequency Data. Appl. Sci. 2020, 10, 8393.

57 Zhang, G.P.; Kline, D.M. Quarterly time-series forecasting with neural networks. IEEE Trans. Neural Netw. 2007, 18, 1800–1814.