

# Fault Diagnosis by Bond Graph Reduced Observer

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**Abstract**—This paper proposes a fault diagnosis method based on reduced observer using the Bond Graph (BG) modeling. Two bank observers structures (BG-DOS) and (BG-GOS) for fault isolation are developed. The effectiveness of the proposed approach is assessed on a real stringing machine.

**Keywords**— diagnosis, reduced observer, Bond Graph, stringing machine.

## I. INTRODUCTION

The fault detection and isolation (FDI) technique have been created due to the required high reliability and the complexity of the dynamical systems. The Luenberger observer-based approach [1], is one of the most famous techniques used for residual generation. When we detect the fault, we need for a single residual. However, the fault isolation requires a set of structured residuals. These residuals must be designed to be sensitive to certain faults and robust to others. In the literature for FDI [2], the bank observers based on analytical models are developed. We found the Dedicated Observer Scheme (DOS) which the  $i^{\text{th}}$  observer is driven by the  $i^{\text{th}}$  output and all inputs. Other outputs are considered unknown, and Generalized Observer Scheme (GOS) which the  $i^{\text{th}}$  observer is driven by all outputs and all inputs except the  $i^{\text{th}}$  output.

Dynamical systems are composed of elements belonging to multiple energy domains (electrical, mechanical, hydraulic, thermal,...). With the causal properties of the Bond Graph (BG) methodology, we can derive the state space form of the system, for fault detection and isolation (FDI) algorithms based on fault indicators generation [3], and design of observers for control [4] and state estimation [5] respectively.

Despite all the contribution related to the BG model yet to date, the diagnosis by BG reduced observer, has not been reported. Our contribution is to extend these observers for fault diagnosis.

This paper is organized as follows: section II proposes a diagnosis by reduced observer using the BG. Section III, an application on the stringing machine is developed and shows the efficiency of the proposed method.

## II. DIAGNOSIS BY REDUCED OBSERVER USING BG APPROACH

### A. Principal of Diagnosis by Reduced Observer

The diagnosis consists on analyzing the residual output estimations  $r_x$  and their sensitivity to faults. Using the observer state estimation, the principle of diagnosis is given in Fig. 1.

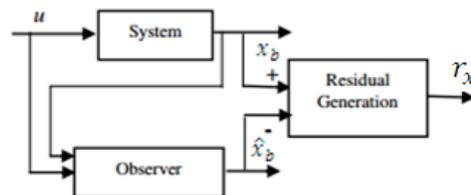


Fig. 1. Diagnosis by reduced observer design

The residual state estimation equation is  $r_x = x_b - \hat{x}_b$

### B. BG Modelling

The BG approach was defined in 1961 by [6] and then developed by [7]. This energetic approach serves to point out analogies between different fields of physics. Due to its structure and causal properties, the BG tool is more and more used for modelling and fault diagnosis.

In our work, the BG is used for modelling, state estimation, fault diagnosis and simulation of dynamical system.

### C. Observer Design by BG Modelling

From a BG point of view, we have to check the system observability, to design the observers. So, according to [8], a BG model is structurally observable if the following two conditions are satisfied:

**First condition:** There is at least a causal path linking actuators (respectively sensors) for each dynamic element  $I$  or  $C$ , in the integral causality when we put the bond graph system model in preferred integral causality.

**Second Condition:** All  $I$  or  $C$  elements derived assuming causality when placed bond graph model in derivative causality, and that the actuators (respectively a sensors) are dualized.

The reduced observer equation using BG variables is shown in eq. (1) :

$$\begin{cases} \dot{\hat{z}} = \bar{M}\hat{z} + \bar{N}u + \bar{P}y \\ \hat{x}_b = \begin{pmatrix} \hat{p}_1 \\ \hat{q}_C \end{pmatrix} = \hat{z} + \bar{L}y \end{cases} \quad (1)$$

With  $\hat{z}$  is the auxiliary variable which avoid the time derivation of the output,  $u$  is the input vector,  $y$  is the output vector,  $\bar{L}$  is the gain of reduced observer to be computed and  $\bar{M}, \bar{N}, \bar{P}$  are constant matrices with appropriate dimensions. With

$$\begin{aligned} \bar{M} &= \bar{A}_{bb} - \bar{L}\bar{A}_{ab}, \bar{N} = \bar{B}_b - \bar{L}\bar{B}_a, \\ \bar{P} &= \bar{A}_{ba} + \bar{A}_{bb}\bar{L} - \bar{L}\bar{A}_{aa} - \bar{L}\bar{A}_{ab}\bar{L} \end{aligned} \quad (2)$$

And

$$\begin{aligned} \bar{A}_{aa} &= (C_a A_{aa} + C_b A_{ba}) C_a^{-1} \\ \bar{A}_{ab} &= (C_a A_{ab} + C_b A_{bb} - \bar{A}_{aa} C_b), \bar{A}_{ba} = \\ & A_{ba} C_a^{-1}, \bar{A}_{bb} = A_{bb} - A_{ba} C_a^{-1} C_b \\ , \bar{B}_a &= C_a B_a + C_b B_b, \bar{B}_b = B_b \end{aligned}$$

are the sub-matrices.

To estimate  $x_b(t)$ , we have to simplify the equation of  $\dot{\hat{z}}(t)$ . So,

$$\dot{\hat{z}}(t) = (\bar{A}_{bb} - \bar{L}\bar{A}_{ab})\hat{z}(t) + (\bar{B}_b - \bar{L}\bar{B}_a)u(t) + (\bar{A}_{ba} + \bar{A}_{bb}\bar{L} - \bar{L}\bar{A}_{aa} - \bar{L}\bar{A}_{ab}\bar{L})y(t) \quad (3)$$

With

$$\varphi = -\bar{L}(\bar{A}_{aa}y(t) + \bar{A}_{ab}(\hat{z}(t) + \bar{L}y(t)) + \bar{B}_a u(t)) \quad (4)$$

The structure of reduced observer BG model is presented in Fig. 2. For diagnosis, we have to determine the residual output estimate  $r_x$ .

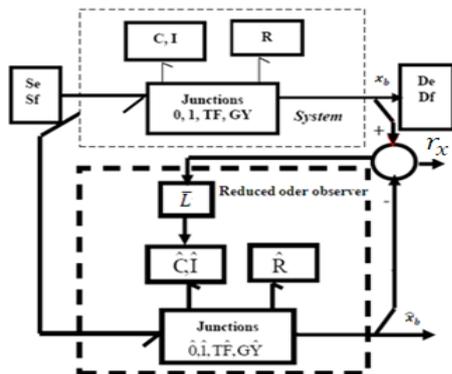


Fig. 2. Structure of reduced observer based on BG

#### D. Strategies of Fault Isolation by Reduced Observer

To isolate the fault by BG approach, we have to extend the bank of observers for FDI sensor modeled by BG approach. Fig. 3 and Fig. 4 represent the BG-DOS and BG-GOS structures needed for fault isolation

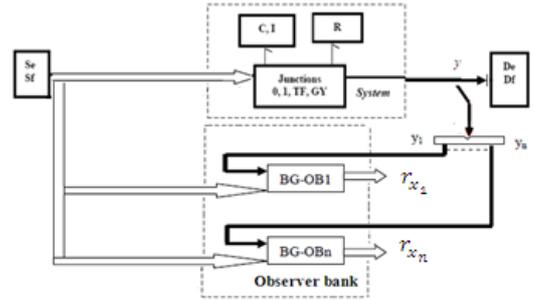


Fig. 3. BG-DOS structure

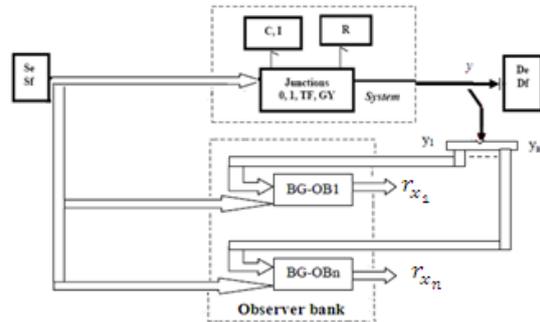


Fig. 4. BG-GOS structure

The obtained residuals deduced from the bank observer are grouped in the FDI table. Its rows and columns correspond to faults and residuals. The table is filled with binary values (fault signature). Zero (0) means that the residual is robust to the fault, and one (1) means that the residual is sensitive.

### III. APPLICATION

#### A. Description of System

We considered here the stringing machine (Fig. 5a). The stringers have used this machine to string rackets for the game professionals (as Roland Garros).

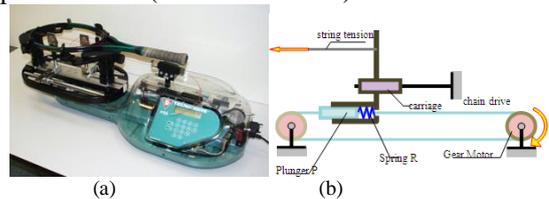


Fig. 5. (a). Stringing machine, (b). Stringing operator modul

The stringing operator module (Fig .5.b) is composed of a gear motor and chain drive. It ensures the movement of the carriage carrying the drawing jaws. The tensioned chain strand is attached to a plunger (P) supported on the carriage by means of a calibrated spring (R). When the stringing operation, the plunger (P) moves to the right relative to the carriage by crushing the spring (R). This displacement is measured by a linear potentiometer which send a signal, the voltage image in the string, to the electric control unit (ECU). The latter then operates the motor control required to the precise string tension.

The word BG of the stringing operator module in open loop is presented in Fig. 6.

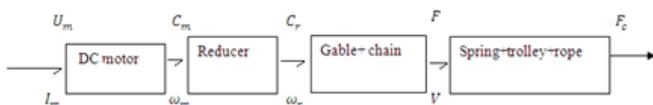


Fig. 6. Word BG of stringing machine

**B. BG Model of Stringing Machine**

The BG modelling of the stringing machine is presented in Fig. 7. The first junction 1 is used to associate the physical phenomenon or components considered by the induced current  $I_m$ . Whether,  $U_m$  is the induced tension,  $R_m$  is the resistance and  $L_m$  is the inductance.

The second junction 1 is used to associate the physical phenomenon or components considered by the mechanic part which depends of the rotation speed of its axe. Whether  $R_1$  is the resistive viscous friction, and  $J_m$  is the moment of the rotor, the shaft and the reducer inertia.

The gyrator element has as  $r_1$  constant, transforms the electromotive force in rotation speed of the shaft of reducer.  $C_1$  is the coefficient of compressibility.

The transformer element has as  $r_2$  constant, transforms the rotation movement in translation movement via the rope winding. The mass of the chain is given by the element  $I = m$  and the frictions at the gable are negligible.

We consider that the tree is of elastic type (whether  $C_2 = \frac{K_r}{1 + K_r/K_c}$ , the loss resistance is given by  $R_2$ ).

The mass of carriage is negligible.

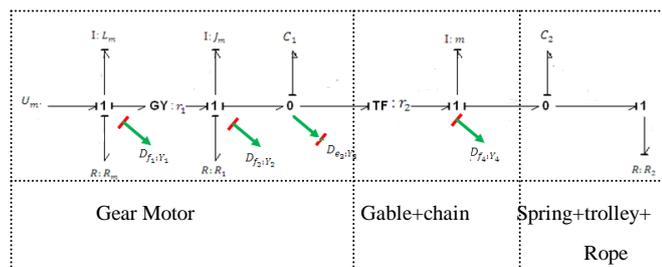


Fig. 7. BG model of stringing machine

From the Fig. 7, we can deduce the state equation [5]:

$$\begin{cases} \begin{bmatrix} \dot{p}_{L_m} \\ \dot{p}_{J_m} \\ \dot{q}_{C_1} \\ \dot{p}_m \\ \dot{q}_{C_2} \end{bmatrix} = \begin{bmatrix} -R_m/L_m & -r_1/J_m & 0 & 0 & 0 \\ r_1/L_m & -R_1/J_m & -1/C_1 & 0 & 0 \\ 0 & 1/J_m & 0 & -1/mr_2 & 0 \\ 0 & 0 & 1/C_1r_2 & 0 & -1/C_2 \\ 0 & 0 & 0 & 1/m & 0 \end{bmatrix} \begin{bmatrix} p_{L_m} \\ p_{J_m} \\ q_{C_1} \\ p_m \\ q_{C_2} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} U_m \\ Y = \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \\ y_4(t) \end{bmatrix} = \begin{bmatrix} 1/L_m & 0 & 0 & 0 & 0 \\ 0 & 1/J_m & 0 & 0 & 0 \\ 0 & 0 & 1/C_1 & 0 & 0 \\ 0 & 0 & 0 & 1/m & 0 \end{bmatrix} \begin{bmatrix} p_{L_m} \\ p_{J_m} \\ q_{C_1} \\ p_m \\ q_{C_2} \end{bmatrix} \end{cases} \quad (5)$$

The parameters of stringing machine are presented in TABLE.I.

TABLE I.  
PARAMETERS VALUE OF STRINGING MACHINE

Symbol	Designations	Nominal Values
$R_m$	Rotor resistance	1.1 $\Omega$
$L_m$	Rotor inductance	1 mH
$J_m$	Moment of geared motor	0.05 $Kg \cdot m^2$
$R_1$	Coefficient of viscous	0.28 N.m/rad/s
$r_1$	Coefficient of torque	0.0386 N.m/A
$r_2$	Reduction ratio	0.01 N.m/A
$m$	Chain mass	0.3 Kg
$K_r$	Spring stiffness	4 N/mm
$K_c$	Rope stiffness	32.7 N/mm
$C_1$	Coefficient of compressibility	$10^{-4}$
$C_2$	Coefficient of compressibility	0.00028
$R_2$	Loss resistance	1000 N.m/rad/s

**C. Diagnosis by Reduced Observer using BG Approach**

We have verified the existence conditions of reduced observer design of the stringing machine modeled by BG:

When we put the BG model of the system with preferred integral causality, there is a causal path linking the sensors  $Df_1(Y_1), Df_2(Y_2), De_3(Y_3)$  and  $Df_4(Y_4)$  for each dynamic elements  $L$  or  $C$ . Also, when the BG model of the system is affected with derivative causality, all the elements  $L$  or  $C$  have derivative causalities and the sensors  $Df_1, Df_2, De_3$  and  $Df_4$  are dualized.

Using the bicausality concept used by [9], the reduced observer design is presented by Fig. 8.

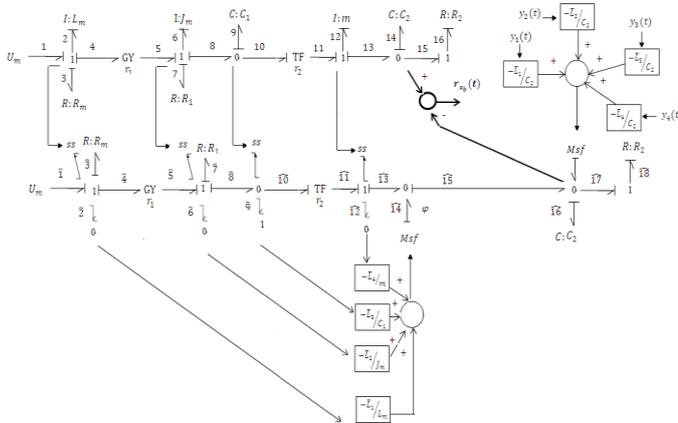


Fig. 8. Reduced observer BG model

We have simulated the system with 20sim. Fig. 9 shows the real and estimated state evolution.

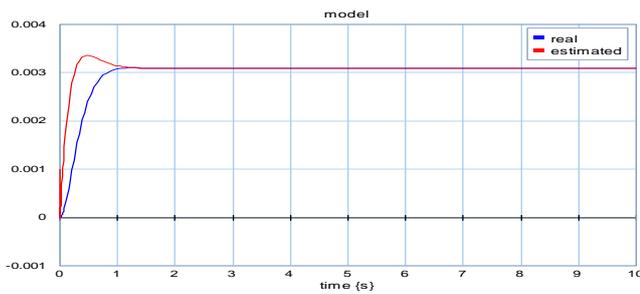


Fig. 9. State variables evolution

1) *Residual generation in normal operation:* From BG model of Fig. 8, we can deduce the residual,  $r_{x_b}(t)$ .

$$r_{x_b}(t) = x_b(t) - \hat{x}_b(t) = e_{14} - \hat{e}_{16} \quad (6)$$

Therefore,

$$e_{14} = \frac{1}{r_2} y_3(t) + m \frac{d}{dt} y_4(t) \quad (7)$$

and

$$\hat{e}_{16} = \hat{e}_{15} = \hat{e}_{14} = \varphi \quad (8)$$

So

$$r_{x_b}(t) = \frac{1}{r_2} y_3(t) + m \frac{d}{dt} y_4(t) - \varphi \quad (9)$$

The Laplace transformation of equation (9) is

$$r_{x_b}(p) = \frac{1}{r_2} y_3(p) + m p y_4(p) - \varphi \quad (10)$$

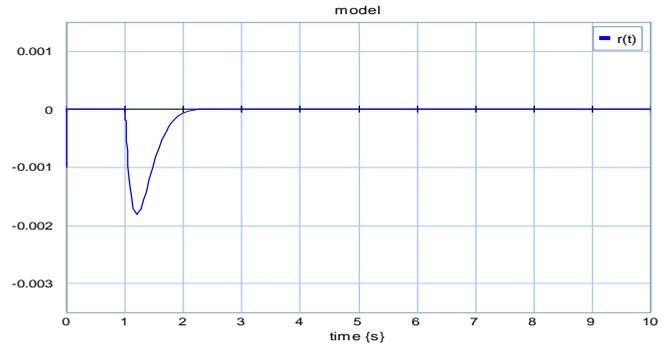


Fig. 10. Residual evolution in normal operation

Fig. 10 shows that the residual converge to zero.

2) *Residual generation with sensors faults:* The sensors

$Df_1, Df_4$  are affected by faults  $f_{c_2}$  and  $f_{c_4}$  from  $t = 4s$  until  $t = 5s$  and from  $t = 8s$  until  $t = 9s$  respectively.

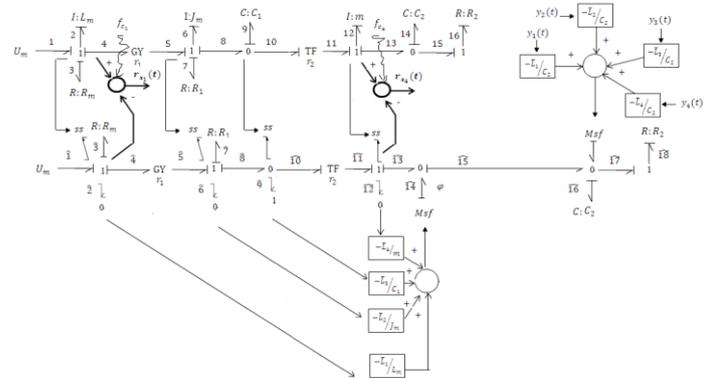


Fig. 11. Reduced observer BG model with sensors faults

From BG model of Fig. 11, we can deduce the residuals,  $r_{x_2}(t)$  and  $r_{x_4}(t)$ .

So,

$$r_{x_1}(t) = (f_2 + f_{c_1}) - \hat{f}_2 \quad (11)$$

With

$$f_2 = \frac{1}{L_m} \int [U_m(t) - R_m(y_1(t) + f_{c_1}(t)) - r_1(y_2(t) + f_{c_2}(t))] dt \quad (12)$$

And

$$\hat{f}_2 = \frac{1}{L_m} \int \left[ U_m(t) - R_m y_1(t) - \frac{r_2}{r_1} y_4(t) + f_{c_1}(t) - r_1 C_1 \frac{d}{dt} y_3(t) \right] dt \quad (13)$$

So,

$$r_{x_1}(t) = \frac{1}{L_m} \int \left[ U_m(t) - R_m (y_1(t) + f_{c_1}(t)) - r_1 (y_2(t) + f_{c_2}(t)) \right] dt - \frac{1}{L_m} \int \left[ U_m - R_m y_1(t) - \frac{r_2}{r_1} (y_4(t) + f_{c_4}(t)) - r_1 C_1 \frac{d}{dt} y_3(t) \right] dt \quad (14)$$

The Laplace transformation of equation (14) is

$$r_{x_1}(p) = \frac{1}{L_m p} \left[ U_m(p) - R_m (y_1(p) + f_{c_1}(p)) - r_1 (y_2(p) + f_{c_2}(p)) \right] - \frac{1}{L_m p} \left[ U_m(p) - R_m y_1(p) - \frac{r_2}{r_1} (y_4(p) + f_{c_4}(p)) - r_1 C_1 p y_3(p) \right] \quad (15)$$

$$\Rightarrow r_{x_4}(t) = (f_{12} + f_{c_2}) - \hat{f}_{12} \quad (16)$$

With

$$f_{12} = y_4(t) + f_{c_4}(t) \quad (17)$$

And

$$\hat{f}_{12} = r_2 \frac{1}{r_1} \left( U_m(t) - L_m \frac{d}{dt} (y_1(t) + f_{c_1}(t)) - R_m y_1(t) - C_1 \frac{d}{dt} y_3(t) \right) \quad (18)$$

So,

$$r_{x_4}(t) = y_4(t) + f_{c_4}(t) - r_2 \left[ \frac{1}{r_1} \left( U_m(t) - L_m \frac{d}{dt} (y_1(t) + f_{c_1}(t)) - R_m y_1(t) - C_1 \frac{d}{dt} y_3(t) \right) \right] \quad (19)$$

The Laplace transformation of equation (19) is

$$r_{x_4}(p) = y_4(p) + f_{c_4}(p) - r_2 \left[ \frac{1}{r_1} \left( U_m(p) - L_m p (y_1(p) + f_{c_1}(p)) - R_m y_1(p) - C_1 p y_3(p) \right) \right] \quad (20)$$

The equations (14) and (19) above show that the residuals are sensitive to sensors faults.

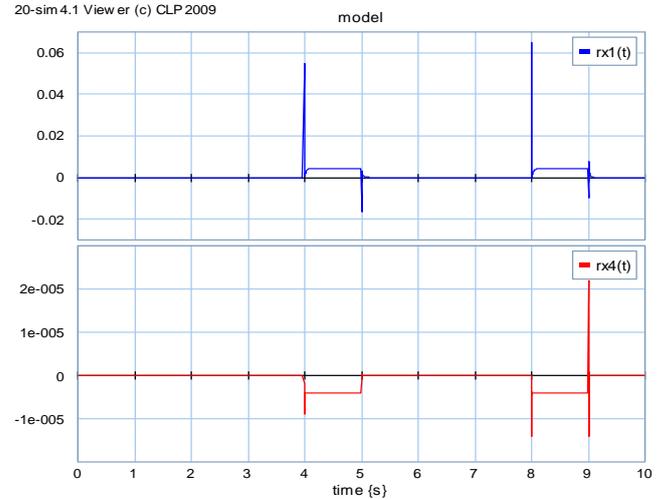
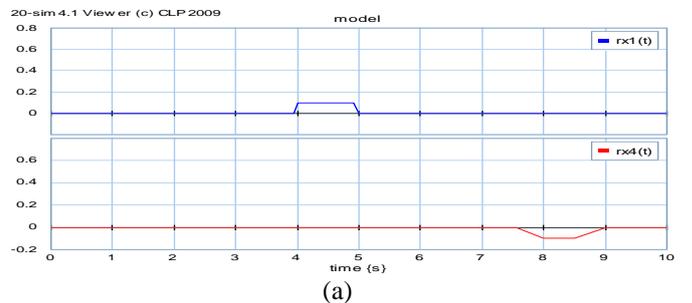


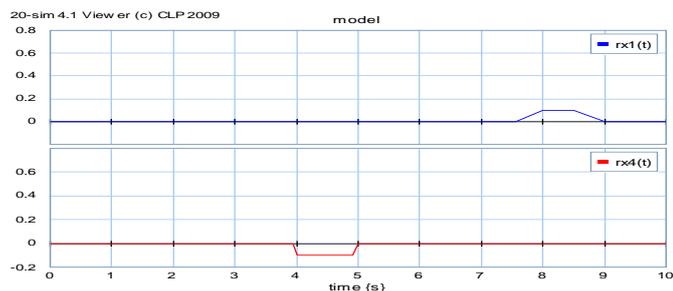
Fig. 12. Residual evolution of  $r_{x_1}(t)$  and  $r_{x_4}(t)$  in faulty operation

The Fig. 12 confirms that the residuals  $r_{x_1}(t)$  and  $r_{x_4}(t)$  are sensitive to faults  $f_{c_2}$  and  $f_{c_4}$ .

3) *Sensor fault detection and isolation:* Fig. 13.a shows the residuals evolution based on BG model using BG-DOS structure. The residual  $r_{x_1}(t)$  is sensitive to the sensor fault  $Df_1$  and it's insensitive to the sensor fault  $Df_4$ . The residual  $r_{x_4}(t)$  is sensitive to sensor fault  $Df_4$ , and it's insensitive to the sensor fault  $Df_1$ .

The Fig. 13.b shows the residuals evolution based on BG model using BG-GOS structure. The residual  $r_{x_1}(t)$  is sensitive to the sensor fault  $Df_4$  and it's insensitive to the sensor fault  $Df_1$ . The residual  $r_{x_4}(t)$  is sensitive to sensor fault  $Df_1$ , and it's insensitive to the sensor fault  $Df_4$ .





(b)

Fig. 13. Residual evolution with observer bank: (a) BG-DOS, (b) BG-GOS

So, the deduced binary signatures can perfectly isolate the fault (TABLE.II).

TABLE II. BINARY SIGNATURES

BG-DOS

BG-GOS

	$Df_1$	$Df_4$
$r_1$	1	0
$r_4$	0	1

	$Df_1$	$Df_4$
$r_1$	0	1
$r_4$	1	0

#### IV. CONCLUSIONS

In this paper, a fault detection and isolation (FDI) based on BG reduced observer bank is proposed. Taking into account the advantage of structural properties of the BG model, the observer is designed graphically. The fault indicators are generated in the presence of sensor faults. The BG-DOS and BG-GOS structures are developed in which way the fault is perfectly isolated. At the last, a particular attention will be paid to the study of the fault estimation based on BG reduced observer in the future research works.

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