

Output waveforms of Blumlein-line Nitrogen Laser Circuit Based on the Distributed Parameter Model: Theoretical and Experimental Results

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Abstract— Optical power calculation of the Blumlein-line nitrogen laser circuit based on the distributed parameter model and the decoupling approach of the laser rate equation from the electrical circuit equations is developed and investigated. The dependence of both the electrical and optical power waveforms on the spark gap inductance is performed and discussed. The saturated laser power approach is assumed in calculating the optical power. The measured laser output waveform obtained is fairly equivalent to the calculated laser power waveform based the distributed model. The theoretical work suggested here could be used to estimate some of the system parameters and help in optimization of the circuit for better system performance.

Keywords— Blumlein-line; Fast discharge laser; Nitrogen laser; Power calculations; optical waveform.

I. INTRODUCTION

Nitrogen lasers are important because they can provide high-power short-duration pulses of ultraviolet radiation ($\lambda = 337.1$ nm). These lasers are widely used in spectroscopy and fluorescence studies, pumping of dye lasers and other research and industrial applications [1]–[6].

The performance of the nitrogen laser is basically determined by the type of electrical system used to create the discharge in the gas. Many excitation schemes were employed for pumping nitrogen lasers, however, the Blumlein transverse excitation method has become very popular, because of its low cost and ease of construction [2]–[8].

The Blumlein-line pulse forming network consists of two parallel plate transmission lines (or coaxial cables) acting as energy storage capacitors, located at both sides of the cavity charged to high voltage V_0 . When one side is short-circuited, for instance using a spark gap, a transient voltage occurs across the laser cavity creating a gas discharge between the electrodes. The spark gap

and the laser gap are usually represented by resistances and inductances.

Depending on the relevant time constants of the spark gap and laser gap along with the wave propagation time on the transmission line, two concepts can be used in the analysis of the Blumlein-line circuit, the lumped parameter model (LPM) and the distributed parameter model (DPM). From a theoretical point of view, the LPM offers the advantage of a much simpler analysis over the DPM., however, it has the disadvantage of being valid only when the relevant time constants of the spark and laser gap are much larger than the wave propagation time on the transmission line [9].

In spite of extensive studies and investigations that have been made so far for understanding the performance of N_2 lasers based Blumlein-line pulse forming network, still extra research work has to be made that include the selection of more accurate circuit models for simulating the laser system and also the effects of the electrical parameters on the overall laser performance.

The N_2 Laser is a highly integrated electro-optical system and the prediction of the behaviour of N_2 laser requires a complicated theory that must include the electric circuit parameters, the gas kinetic parameters and detailed mechanism of energy transfer of the three laser levels of the molecular nitrogen. A significant simplification of the calculation has been brought about by Fitzsimmons [10]. Instead of estimating the electron temperature by solving the energy balance equation as a part of the coupled system, they used, in a decoupled procedure, another alternative approach to predict the electron temperature. Within this framework of the assumptions, the

electron temperature is determined in terms of the instantaneous value of ratio of the electric field between the laser channel electrodes divided by the pressure inside the laser channel. Hence this procedure makes it possible to handle the laser rate equations in separate way. This can be done after getting the required time development of the electric field from the solution of the electric circuit equations. In the previous work, the general laser power assumption was used in calculating the output optical power, and the influence of the laser gap inductance (laser channel inductance) on the laser output power waveforms were investigated and studied [11].

This paper reports the theoretical analysis, power calculations and the effect of the spark gap inductance on the output waveforms of the Blumlein-line N_2 laser circuit. Here, the simplified saturated laser power approach was used in calculating the output optical power. A homemade Blumlein-line circuit was constructed and built with a free running spark gap. The measured current waveform was obtained by using a homemade current probe.

II. TRANSMISSION LINE EQUATIONS OF THE BLUMLEIN LINE AND THE BOUNDARY CONDITIONS

A mathematical configuration of the Blumlein circuit is shown in Fig.1. The Blumlein line is divided into two sections, the right hand section and the left hand section with different zero coordinates. The applicable Transmission line equations for the voltage V and current I on a section of the transmission line of dx at any time t are given by [9],

$$\partial V_i(x_i, t) / \partial x = -\bar{L} \partial I_i(x_i, t) / \partial t \quad (1)$$

$$\partial I_i(x_i, t) / \partial x = -\bar{C} \partial V_i(x_i, t) / \partial t \quad (2)$$

$i =$ left and right, $0_i \leq x_i \leq l_i$, \bar{L} and \bar{C} are the distributed inductance and capacitance per unit length respectively, and l is the total length of line section, Z_0 ($z_0 = \sqrt{\bar{L}/\bar{C}}$) is the characteristic impedance of the line. The initial conditions are:

$$V_i(x_i, 0) = V_0 \quad (3)$$

$$I_i(x_i, 0) = 0 \quad (4)$$

The boundary condition at the end of the spark-gap is:

$$V_R(0_R, t) = -R_S [I_R(0_R, t) + \frac{L_S}{R_S} \partial I_R(0_R, t) / \partial t] \quad (5)$$

The boundary conditions at the end of the channel are:

$$V_R(l_R, t) - V_L(0_L, t) = L_g \partial I_L(0_L, t) / \partial t + R_g I_L(0_L, t) \quad (6)$$

$$I_R(l_R, t) = I_L(0_L, t) \quad (7)$$

The boundary condition at the open end is:

$$I_L(l_L, t) = 0 \quad (8)$$

where (L_S and R_S) and (L_g and R_g) stand for inductances and resistors of the spark gap and laser gap, respectively. The laser gap impedance is not linear and must be treated as voltage dependent: as an open circuit before laser gap breakdown and represented by a combination of (L_g and R_g) after laser gap breakdown. I_R is the current passed in the spark gap inductance and I_L is the current passed in the channel inductance after laser gap breakdown. This nonlinear behavior of the channel impedance, makes it necessary, in the time development of solution of the partial differential equation (6) to distinguish between two time intervals: the first one is before laser gap breakdown, during which the state of only the right-hand section is subject to the dynamical changes, whereas the left-hand section remains in its initial state, and the second time interval is after laser gap breakdown, in which the states of both sides are time varying. The subscripts L and R in the current, voltage and lengths denoting for the left hand and the right hand sides of the Blumlein-line.

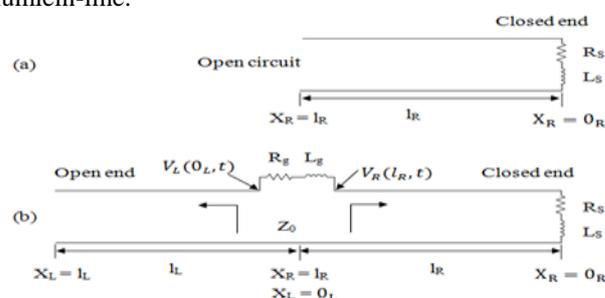


Fig. 1. Representation of Blumlein N_2 Laser circuit. (a). Before breakdown (b). After breakdown.

III. SYSTEM OF EQUATIONS FOR OPTICAL POWER COMPUTATION

Calculation of the optical power is performed under the saturation approximation assumption. This method assumed that the laser transition is saturated; i.e., the laser power, P_0 is strong enough to equalize the densities N_C and N_B in the C and B states energy laser levels of the molecular nitrogen respectively. We can then let $N_C - N_B \ll N_C$ and set $N_C = N_B = N$ and then calculate P_0 . These equations can be easily derived in term of the instantaneous

electric field E and the pressure P inside the laser tube and written as:

$$\frac{dN}{dt} + \frac{N}{2\tau_B} = \frac{1}{2} \left[n_e N_o (\sigma_{OB} + \sigma_{OC}) \left(\frac{0.88}{m\pi} \right)^{\frac{1}{2}} \left(\frac{E}{P} \right)^{\frac{2}{5}} \right] \quad (9)$$

$$P_o = \frac{1}{2} \left[n_e N_o (\sigma_{OC} - \sigma_{OB}) \left(\frac{0.88}{m\pi} \right)^{\frac{1}{2}} \left(\frac{E}{P} \right)^{\frac{2}{5}} - \frac{2 - \left(\frac{\tau_C}{\tau_B} \right)}{\tau_C} N \right] \quad (10)$$

E is given by $[V_R(l_R, t) - V_L(0_L, t)]/d$. d is the electrodes separation. n_e is the electron density, $N_o (\cong 3.2 \times 10^{16} P)$ is the ground-state density which is assumed constant, τ_C is the radiative lifetime of the C-state (40ns) and τ_B is the radiative lifetime of the B-state (10 μ s). σ_{OC} and σ_{OB} are the electron impact cross sections of the C and B states ($11.1 \times 10^{-18} \text{cm}^2$ and $9.2 \times 10^{-18} \text{cm}^2$ respectively). m is the electron mass ($9.2 \times 10^{-28} \text{g}$). The above equations can be solved after deriving the electron density equation in terms of (E/P) to obtain n_e inside the laser channel as a function of time. The electron density equation can also be derived and written as:

$$\frac{dn_e}{dt} = 4.06(10^{-3}) \left(\frac{E}{P} \right)^{4.7} P \cdot n_e \quad (11)$$

IV. SELECTION OF THE SCHEMES AND COMPUTATIONAL PROCEDURE

To obtain voltage and current solutions across the laser channel, we applied numerical schemes that based on the finite difference method, the forward, backward and central difference schemes along-with the "Lax-wendroff" scheme" to the partial differential equations of the Blumlein-line and the boundary conditions. The method and the schemes used were presented in detail in the previous work [9]. Standard numerical techniques were used in solving the excitation rate equation and the electron density equation.

V. COMPUTATIONAL RESULTS AND DISCUSSIONS

Following the application of the numerical schemes mentioned in (III) and simulation of the laser system, useful results were obtained. Fig. 2 shows the waveform of the instantaneous (E/P) a cross the laser channel and influence of the spark-gap inductance on $\partial(E/P)/\partial t$, whereas the electrical current, electrical power and optical power waveforms along-with the effect of the spark-gap inductance on them are shown in Figs. 3-5.

The values of circuit elements used in the simulation are: $\bar{C}=128.88\text{nF/m}$, $\bar{L}=0.288\text{nH/m}$, $R_g=0.1\Omega$, $R_s=0.0366\Omega$, $L_g=0.2\text{nH}$, $l_L=55\text{cm}$ (open end side) and $l_R=50\text{cm}$ (spark gap side). The chosen values of L_s for the simulation are 14nH and 28nH. The electrodes separation used in the simulation is 1.5cm whereas their lengths and widths are 30cm and 0.5cm respectively. The pressure inside the laser tube is 67Torr. The electrical power is obtained from the expression $(I^2 R_g)$ whereas the optical power is obtained from (10). It is clear from Figs. 2-5 that the laser gap breakdown is a highly dependent on the spark gap inductance. Therefore, in order to

obtain a smaller breakdown time in the gas, a smaller value of the spark-gap inductance must be maintained.

In the waveform figures, the time is normalized by $T_{normalized} = \frac{time}{l_R \sqrt{LC}}$

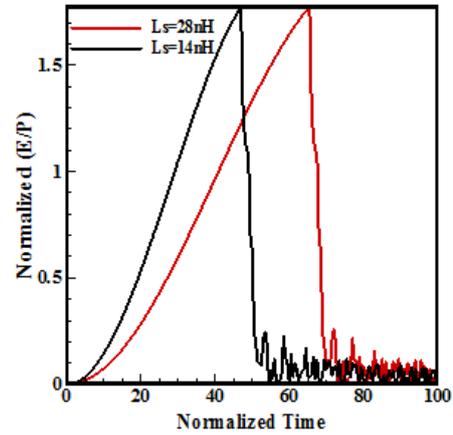


Fig. 2. Instantaneous E/P waveform and the influence of the spark-gap inductance on $\partial(E/P)/\partial t$ a cross the laser channel.

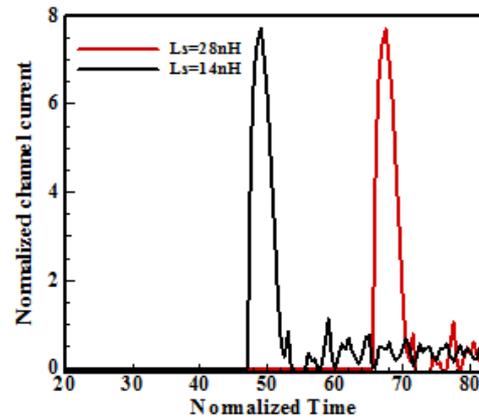


Fig. 3. Influence of the spark-gap inductance on the laser channel current

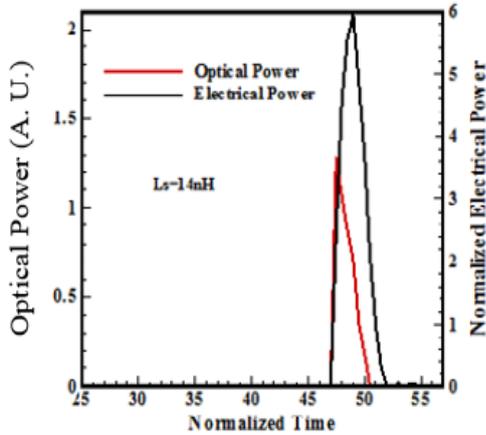


Fig. 4. Influence of the spark-gap inductance on the electrical and optical power waveforms of the laser channel at $L_s = 14\text{nH}$.

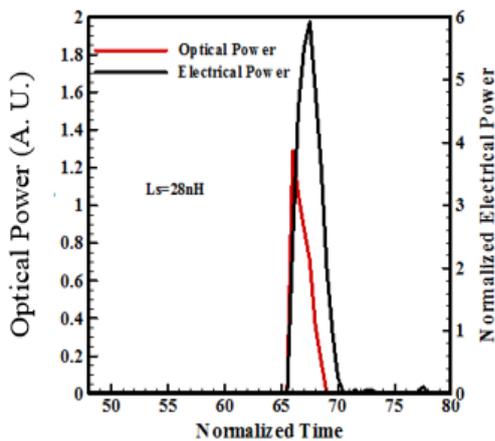


Fig. 5. Influence of the spark-gap inductance on the electrical and optical power waveforms of the laser channel at $L_s = 28\text{nH}$.



(a)



(b)

Fig. 6. (a) Physical construction of the Blumlein N_2 laser (b). The laser tube along-with the output of a nitrogen laser is in the ultraviolet range

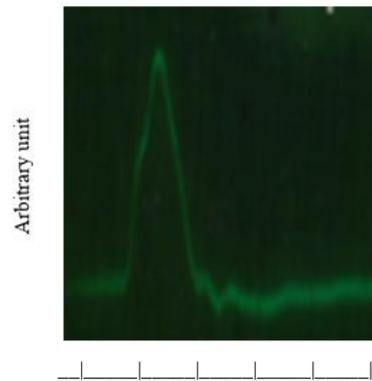


Fig. 7. Oscilloscope trace of photodiode signal. Time base is 10ns/div.

VI. TEMPORAL MEASUREMENT OF THE LASER OUTPUT WAVEFORM

Fig. 6(a) shows the physical construction of the Blumlein N_2 laser system showing the locations of the spark gap and laser tube relative to the copper parallel plates that comprise the energy storage capacitors and Blumlein transmission line.

The parallel plate transmission line are formed from a double sided copper coated printed board 104 x 34 cm, and of 0.1cm thick with relative permittivity of 5.03. Both surfaces of the board were etched away in 2cm wide border to prevent conduction between the plates. The aluminium laser channel electrodes are 30cm in length with a thickness of 0.1cm where they are separated by 1.2cm. The spark gap used was a free running two electrodes spark gap. The spark gap electrodes formed from two aluminium half spheres of 2cm in diameter. The spark gap is designed such that the distance between the electrodes can be controlled. Fig. 6(b) shows the laser tube along-with the output of a nitrogen laser is in the ultraviolet range, at a wavelength of 337.1nm, thus invisible. However, its beam can be observed as a light blue spot with a

fluorescent material such a pieces of paper or cardboard placed at distance along the laser channel. The pulse shape of the laser output was detected by using OR-7184 photo diode together with a Teck.475 Oscilloscope. The pulse shape is shown in Fig7. The measured pulse width (FWHM) is 11nsec and it is supposed to be around 6nsec (typical values of such lasers), because we used a photodiode with limited rise time of 6.8nsec. It is clear that the measured and simulated optical power waveforms almost have the same form. This could help in estimating the laser channel parameters.

VII. CONCLUSIONS

A full distributed parameter model of the Blumlein-line N_2 laser with the decoupling approach of the laser rate equations from the electrical circuit equations has been developed and investigated. The effect of the spark-gap inductance on the instantaneous (E/P), laser channel current, electrical power and optical power waveforms were carried out. The investigation results show that the time of ignition (the gas breakdown) depends on the spark gap inductance, i.e., the less the value of the spark-gap inductance the faster gas breakdown will occur, which is also very clear in the shift occurred in the current waveforms. With the use of this modified analysis, the N_2 laser system may easily be optimized over a wide range of parametric variations that could assist in optimization efforts. This is because the distributed parameter model used here were proved to be more accurate than the other lumped parameter models in the previous work, which will provide a better predictions for the behavior of N_2 laser that are needed for better system performance and optimization. The analysis presented here is quite general and could be applied to many other gas laser systems. The experimental measurement of the laser output waveform presented here along-with the theoretical simulations might help in estimating the laser channel parameters that could help, in system optimization and design verification. Further experimental and theoretical work will be pursued for extra experimental verifications.

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