

Contribution to the Maintenance of Manufacturing Systems with Time Constraints Using Fuzzy Petri Nets

Anis MHALLA

National School of Engineering of Tunis, BP 37, le Belvédère,
1002 Tunis, Tunisia
anis.mhalla@enim.rnu.tn

Mohamed BENREJEB

National School of Engineering of Tunis, BP 37, le Belvédère,
1002 Tunis, Tunisia
mohamed.benrejeb@enit.rnu.tn

Abstract— The aim of this paper is the study and the design of a maintenance module based on Petri nets (PNs) for manufacturing job-shops with time constraints. In such systems, operation times are included between a minimum and a maximum value. In this context, we propose a new fuzzy Petri net called Fuzzy Petri Net for maintenance (FPNM). This tool is able to analyze monitoring and recovery tasks of a discrete event system with time constraints, using a temporal fuzzy approach. The maintenance module is consists of P-time PNs and fault tree. The first is used for modelling of normal behaviour of the system by temporal spectrum of the marking. The second model corresponds to diagnosis activities. Finally, to illustrate the effectiveness and accuracy of proposed maintenance approach, two industrial examples are depicted.

Keywords— Fuzzy numbers; P-time PN; Fault Tree; Alpha-cut; FPNM; Recovery.

I. INTRODUCTION

The demands for products with higher quality and competitive prices have led to the development of complex manufacturing systems. A consequence is that the number of failures tends to increase as well as the time required to locate and repair them. The occurrence of failures during nominal operation can deeply modify the flexible manufacturing systems (FMS's) performance or its availability [1]. Thus it is imperative to implement a maintenance strategy allocated to the FMS's.

In this paper, we propose a new maintenance approach based on the study of effective sojourn time of the token in places and the evaluation of the "failure probability of the top event", in manufacturing systems with staying time constraints. In the category of the workshops concerned by this paper, the operations have temporal constraints which must be imperatively respected. The violation of these constraints can affect the health of the consumers. Thus, the detection of a constraint violation must automatically cause the stop of the production. Maintenance and its integration with control and monitoring systems, enable the improvement of manufacturing systems, regarding availability, efficiency, productivity and quality [1]. Thus, it is possible to implement corrective and preventive actions in manufacturing systems with time

constraints in order to make repairs and servicing easier over the process elements, as well as a better control provision of tools and repair parts.

This paper is organised as follows. The second section begins by presenting the formal definition of P-TPN as a modelling tool and summarizes the classes of uncertainties in manufacturing workshops with time constraints. Section 3, introduce the fuzzy probabilistic approach to evaluate failure probability of the top event, when there is an uncertainty about the components failure probabilities. Afterward, the problem of maintenance of manufacturing systems is tackled. An original recovery approach based on PN's, is presented.

In Section 5, two academic examples (workshops with/without assembling tasks) are then used to illustrate the different steps of the proposed approach. Finally, a conclusion is presented with some perspectives

II. REPRESENTATION OF UNCERTAINTY IN MANUFACTURING WORKSHOPS

A. P-time Petri nets

From the modelling point of view, P-TPNs were introduced in 1996 in order to model Dynamic Discrete Event System (DDES) including sojourn time constraints.

Definition 1 [2]: The formal definition of a P-TPN is given by a pair $\langle R; I \rangle$ where:

R is a marked Petri net,

$$I : P \rightarrow Q^+ \times (Q^+ \cup \{+\infty\})$$

$$p_i \rightarrow IS_i = [a_i, b_i] \text{ with } 0 \leq a_i \leq b_i.$$

IS_i defines the static interval of staying time of a mark in the place p_i belonging to the set of places P (Q^+ is the set of positive rational numbers). A mark in the place p_i is taken into account in transition validation when it has stayed in p_i at least a duration a_i and no longer than b_i . After the duration b_i the token will be dead.

In manufacturing job-shops with time constraints, for each operation is associated a time Interval ($[a_i, b_i]$ with u.t: unit

time). Its lower bound indicates the minimum time needed to execute the operation and the upper bound sets the maximum time not to exceed in order to avoid the deterioration of the product quality. Consequently P-TPNs have the capability of modelling time intervals and deducing a set of scenarios, when time constraints are violated.

B. Uncertainty in manufacturing workshops

The production is subject to many uncertainties arising from the processes, the operators or the variations of quality of the products. A production is seldom perfectly repetitive. All authors, who treated uncertainties, studied mainly two disturbances: disturbances on the equipment and more particularly the breakdowns machine or the disturbances concerning work and more particularly the change in the operational durations [3]. For all these reasons, a function of possibilities, representing uncertainty over the effective residence time (q_i) of a token in a place p_i , is proposed. This function makes it possible to highlight zones of certainty for an operational duration and helps the human agent (or supervisor) in charge of detecting failures and deciding reconfiguration/repair actions [4].

1) Graphical representation of effective sojourn time uncertainty

In order to quantify to a set of possible sojourn time of the token in the place p_i , a fuzzy set A , representing the uncertainty on the effective sojourn time of the token in the place p_i (q_i) is proposed (Figure 1).

This quantification allows us to define a measure of the possibility with which the sojourn time q_i , is verified. These results, Figure 1, make it possible to highlight zones of certainty for operation durations; a high value of effective sojourn time can guarantee a normal behaviour of monitored system. Instead, a low value implies the possibility of detecting of failure symptom (behavioural deviation).

Based on fuzzy model, Figure 1, all system scenarios are developed. The scenarios consider all possible deviations. Deviations can occur due to the failure of components. Then from fuzzy model, we deduce a set of scenarios (events sequences) bringing the system to erroneous situations (failure).

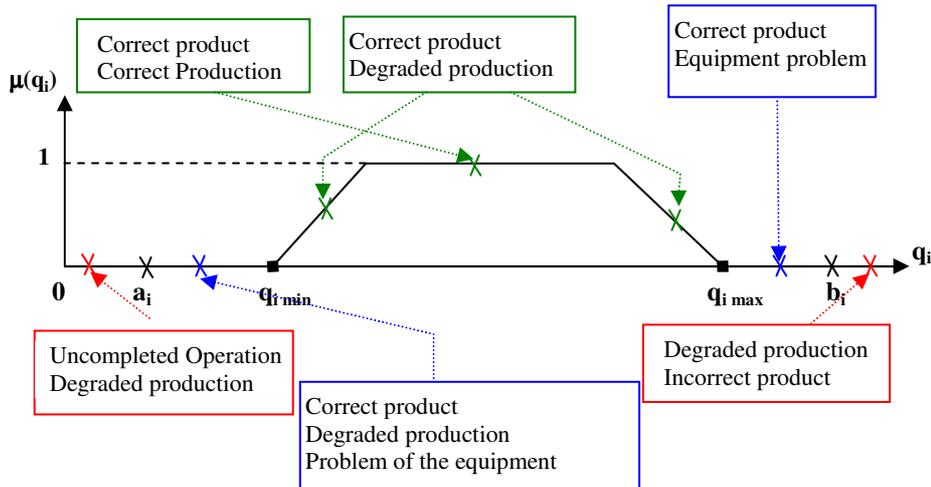


Fig.1. Function of possibility associated with an effective sojourn time (q_i)

2) Constraints violation and Recovery task

In manufacturing workshops with time constraints, the fuzzy model associated to effective sojourn time (q_i), monitors the system evolutions through the time durations verification (operating or transfer durations for example) [5]. These durations represent interval constraints. When the interval constraints are exceeded, there is an error.

An error is defined as a discrepancy between an observed or measured value and the true or theoretically correct value or condition [6]. In our study, an error means a gap between measured and computed time intervals by the scheduling task.

Based on the above statements, an error is sometimes referred to as an incipient failure [6]. Therefore maintenance action is taken when the system is still in an error condition, i.e. within acceptable deviation and before failure occurs. Thus, this study employs uncertainty of sojourn time in order to perform early failure detection.

III. ESTIMATION OF FAILURE PROBABILITY BY FUZZY FAULT TREE ANALYSIS

A. Preliminary definitions

Definition 2 [7]: A fault tree FT is a directed acyclic graph defined by the tuple $\{E_i, G_i, D_i, TOP_i\}$. The union of the sets G_i (logical gates) and E_i (events) represents the nodes of the graph ; D_i is a set of directed edges, each of which can only connect an event to the input of a logical gate or the output of a logical gate to an event.

A top event TOP_i is an event of the fault tree FT_i that is not the input of any logic gate, i.e. there are no edges that come out of the top event. The nodes of a fault tree are connected through logical gates, in this paper; we consider only static fault trees, i.e. fault trees in which the time variable does not appear. Therefore, only the AND and the OR gate will be treated in this paper.

Definition 3 [7]: Let us suppose AND_i is an AND gate with n inputs IN_kAND_i , $1 < k < n$ and output $OUTAND_i$.

Let $P_{in}(k, i)$ be the probability associated with the input IN_kAND_i and P_{OUTAND_i} be the probability associated with the output of AND_i .

If the inputs to the AND gate is mutually independent, the probability associated with the output can be calculated as follows:

$$P_{OUTAND_i} = \prod_{k=1}^n P_{in}(k, i) \quad (1)$$

Definition 4 [7]: Let us suppose OR_i is an OR gate with n inputs IN_kOR_i , $1 < k < n$ and output $OUTOR_i$. Let $P_{in}(k, i)$ be the probability associated with the input IN_kOR_i and P_{OUTOR_i} be the probability associated with the output of OR_i .

If the inputs to the OR gate all mutually exclusive, the output can be calculated as follows:

$$P_{OUTOR_i} = 1 - \prod_{k=1}^n (1 - P_{in}(k, i)) \quad (2)$$

B. Fuzzy approach for uncertainty analysis

The fuzzy probabilistic approach aims to quantitatively evaluate the reliability of manufacturing workshops with time constraints. But, as mentioned previously, studies are under uncertainty. The goal of the paper is to take into account these uncertainties in the evaluation. So, we investigate the use of the fuzzy set theory to determine the probability of the top event of the fault tree associated to workshops with time constraint.

1) Fuzzy Numbers

Let x be a continuous variable restricted to a distribution function $\mu(x)$, which satisfy the following assumptions [8]:

- $\mu(x)$ is a piecewise continuous,
- $\mu(x)$ is a convex fuzzy set,
- $\mu(x)$ is a normal fuzzy set.

A fuzzy set which satisfies these requirements is called a fuzzy number. For any fuzzy number \tilde{A} which has the membership function $\mu_{\tilde{A}}(x)$, an interval bounded by two points at each α -level ($0 \leq \alpha \leq 1$) can be obtained using the α -cut method [9]. The symbols $A_L^{(\alpha)}$ and $A_R^{(\alpha)}$ have been used in this paper to represent the $\mu_{\tilde{A}}(x)$ left-end-point and right-end-point of this interval.

As it is shown in figure 2, we can express a fuzzy number, using the following form [10]:

$$\tilde{A} \rightarrow [A_L^{(\alpha)}, A_R^{(\alpha)}] \text{ with } 0 \leq \alpha \leq 1$$

For each α -level of the fuzzy number which represents a probability, the model is run to determine the minimum and maximum possible values of the output. This information is then directly used to construct the corresponding membership function of the output.

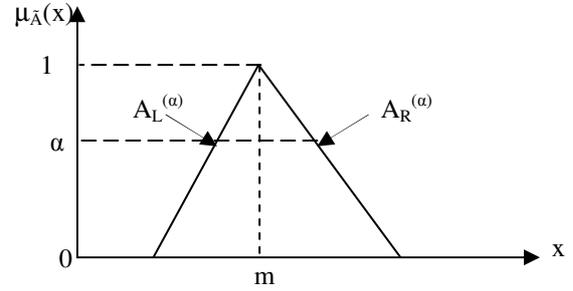


Fig. 2. Bounds points for α -level set interval of $\mu_A(x)$ [8]

2) Fuzzy Probabilities

A fuzzy probability is represented by a fuzzy number between 0 and 1 assigned to the probability of an event occurrence [9], [11], [12].

Our goal is to use fuzzy probabilities to describe occurrence probabilities of events. To this end, we follow the standard approach proposed by Buckley to describe the probabilities of various unions and intersections of these events occurrences.

3) Buckley Approach

An extension of traditional approaches to take account of vagueness is proposed by Buckley [13], [14].

The Buckley approach, associate to each input variables a fuzzy number and combine them sequentially by using the concept of α -cut which reduces the problem to a calculation interval.

Let us consider two fuzzy numbers \tilde{X} and \tilde{Y} , respectively represented by the two intervals $[X_L^{(\alpha)}, X_R^{(\alpha)}]$ and $[Y_L^{(\alpha)}, Y_R^{(\alpha)}]$. Arithmetic operations applied to intervals give the following expressions [13]:

$$\tilde{Z} = \tilde{X} + \tilde{Y} \rightarrow [Z_L^{(\alpha)}, Z_R^{(\alpha)}] = [X_L^{(\alpha)} + Y_L^{(\alpha)}, X_R^{(\alpha)} + Y_R^{(\alpha)}] \quad (3)$$

$$\tilde{Z} = \tilde{X} \cdot \tilde{Y} \rightarrow [Z_L^{(\alpha)}, Z_R^{(\alpha)}] \quad (4)$$

with:

$$\begin{cases} \tilde{Z}_L^{(\alpha)} = \min(X_L^{(\alpha)} \cdot Y_L^{(\alpha)}, X_R^{(\alpha)} \cdot Y_L^{(\alpha)}, X_L^{(\alpha)} \cdot Y_R^{(\alpha)}, X_R^{(\alpha)} \cdot Y_R^{(\alpha)}) \\ \tilde{Z}_R^{(\alpha)} = \max(X_L^{(\alpha)} \cdot Y_L^{(\alpha)}, X_R^{(\alpha)} \cdot Y_L^{(\alpha)}, X_L^{(\alpha)} \cdot Y_R^{(\alpha)}, X_R^{(\alpha)} \cdot Y_R^{(\alpha)}) \end{cases}$$

4) Fault Tree Analysis and maintenance task

Reliability and life are two major elements of maintenance tasks. Reliability theory can also assists maintenance engineers in judging the operational status of equipment and in developing safe response measures to prevent any accidents during shutdown procedures [15]. Such the FTA provides a

basis for further maintenance of manufacturing systems with time constraints, there by, enabling engineers to conduct additional tests to determine a proper reliability distribution model for further analysis and application.

According to the failure records of well-documented manufacturing systems, maintenance tasks are generally categorized as adjustment, repair, and replacement [15].

While failure distribution characteristics are analyzed using fault tree analysis, failure distribution modes and Weibull distributions, are incorporated into a system reliability model and then tested and analyzed to establish a proper reliability distribution model.

IV. FUZZY PETRI NETS FOR MAINTENANCE (FPNM)

In manufacturing system, failure will occur if the degradation level exceeds the permissible value. Therefore, maintenance is defined as a strategy to maintain available or operational conditions of a facility using all possible methods and means, or to restore functions from trouble and failures [1].

Much of development works has been undertaken in certain of the maintenance fields. Recovery tools have been researched [15] and their application to failure prevention is well reviewed [16]. The proposed recovery tool is inspired of the research of E.Minca [16].

To model the recovery functions, a definition of a fuzzy PN model able to integrate uncertainty on sojourn time (q_i) and fuzzy probabilities of the monitored system (P_i), related to a base of fuzzy logic rules, is given, figure 3.

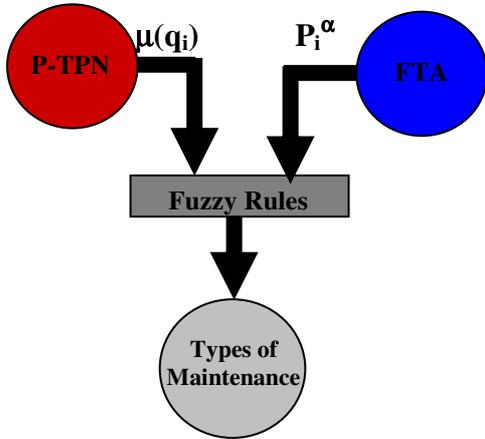


Fig. 3. FPNM structure

A. Definition of FPNM

The fuzzy Petri net for maintenance (FPNM) is defined as being the n-uplet : $\langle P, T, Pr, Q, RA, F, \Psi, \Omega, \Delta, M_0 \rangle$ with:

$P = p^x \cup p^y$: the finite set of input p^x and output p^y places;

$T = \{t_1, t_2, \dots, t_n\}$: a collection of transitions. A transition t_i is specialized in inference/aggregation operations of logic rules;

$Pr = \bigcup_{e=1}^z Pr_e$: the finite set of the input variable “fuzzy probability”;

$Q = \bigcup_{f=1}^r q_f$: subsets of input variables “sojourn time”;

$RA = \bigcup_{g=1}^s ra_g$: subsets of output variables “recovery action”;

Pr (resp Q) and RA are subsets of variables that are respectively in the antecedence and in the consequence of the fuzzy rules F_w ;

$F = \bigcup_{w=1}^\alpha F_w$: $F_w = Pr \cup Q \rightarrow RA$: the fuzzy logic rules set.

$\Psi = (\Psi_{11}, \Psi_{12}, \dots, \Psi_{z1}, \Psi_{z2}, \dots, \Psi_{ze})$: the finite set of membership functions, defined on the universe $[0,1]$ of the input variables “fuzzy probability”, $Pr = (Pr_1, Pr_2, \dots, Pr_z)$. “ e ” represents the number of input variables Pr ;

$\Omega = (\Omega_{11}, \Omega_{12}, \dots, \Omega_{r1}, \Omega_{r2}, \dots, \Omega_{rr})$: the finite set of membership functions, definite on the universe $[0,1]$ of the second input variable “sojourn time”. “ r ” is the number of input variables “sojourn time”;

$\Delta = (\Delta_{11}, \Delta_{12}, \dots, \Delta_{s1}, \Delta_{s2}, \dots, \Delta_{ss})$: the finite set of membership functions, definite on the universe $[0,1]$ of the output variable “recovery action”. “ s ” is the number of output variables;

M_0 : the initial marking of the input places $p_i \in P^x$

Each input or output place of the FPNM is associated to a fuzzy description. For the input places, we describe the marking variable of the place, whereas for the output places we describe recovery action. In FPNM, each base of logic rules “ F ” represents the fuzzy implications describing the knowledge base of the expert.

Each implication respects the “if-then” model represents the logical dependence of variable Pr (resp. Q and RA) associated to the fuzzy sets Ψ (resp. Ω and Δ). The proposed FPNM is considered as an adaptive technique dedicated to the recovering of manufacturing systems with time constraints.

This model has a double interface, one with the modelling model (based on P-time PN) and the second one with the diagnosis model (Fuzzy Fault Tree). To demonstrate the effectiveness of our proposed methodology we present two maintenance realistic examples.

V. ILLUSTRATIVE EXAMPLES

A. Packaging unit (Job Shop with Assembling Tasks)

1) Presentation of packaging unit

For simplicity, we disregard the nature of the precise operations performed in the packaging unit; therefore we represent a simplified model of the unit.

Figure 4, shows a milk packaging unit: to packing the products (bottles of 1000 ml), bottles are placed on the conveyor T_1 to supply the packaging machine (M), where they will be wrapped by welding in a group of 6. The finished products are deposited on the output conveyor towards the stock of finished products SA.

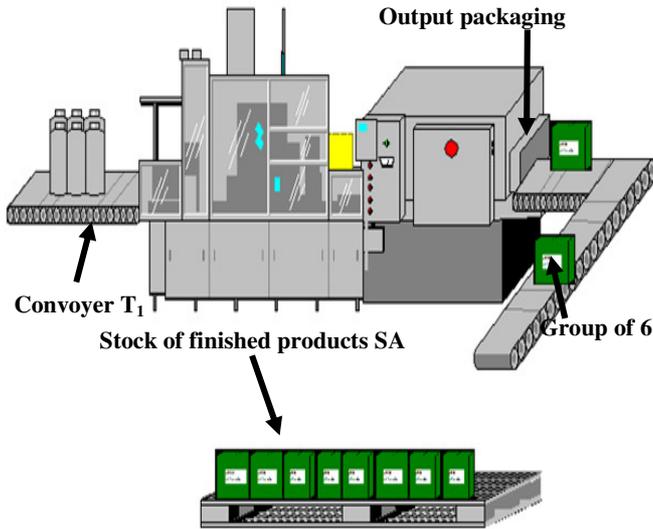


Fig. 4. Packaging machine

2) Modeling of packaging unit

Figure 5, shows a P-time Petri net (G) modeling the packaging machine. Three fuzzy sets, representing the uncertainty on the effective sojourn time of the token in the places p_1 , p_2 and p_8 , are proposed (Figure 5). The obtained membership's functions are used to study the maintenance of the machine (M).

As the sojourn times in places have not the same functional signification when they are included in the sequential process of a product or when they are associated to a free resource, a decomposition of the Petri net model into two sets is made using [17], figure 5, where:

- R_U is the set of places representing the used machines,
- $Trans_C$ is the set of places representing the loaded transport resources.

In milk manufacturing workshop, the operations have temporal constraints which must be imperatively respected. The violation of these constraints can affect the health of the consumers and can induce some catastrophic consequences (inconsumable product, burnt milk ...) [18]. Therefore, the

considered recovering approach uses the additional information provided by the knowledge of interval constraints and by the detection a failure symptom. Let us suppose that we want to monitor the duration of packaging of bottles. According to P-TPN, figure 5, the minimum time granted to the operation is 12 *u.t*, whereas the maximum time is 20 *u.t* ($IS = [12, 20]$; $q_c = 15$). A delay of packaging operation may involve:

- A technical failure of the production tool (conveyor problem for example) requiring to generate a maintenance action,
- The production cycle of a milk bottle can be delayed; in fact a delay can imply the propagation of the failure symptom and can induce some catastrophic consequences on the functioning of the system.

3) Diagnosis of packaging machine

When a constraint is violated a diagnosis text is generated. The diagnosis text determines failed states (deviations from the normal function) of the packaging machine and its subsystem (failure of sealing bar, failure of fingers ...).

When a symptom is claimed, it is imperative to localize the failure by using fault tree as a modeling tool, Figure 6. The logical expression of top event (F) of the fault tree is:

$$F = ds_1 + ds_2 = (a + b) + (c \times d)$$

4) Fuzzy Probabilistic Approach

The Fault Tree analysis (FTA) is based on the fuzzy set theory. So, we can allocate a degree of uncertainty to each value of the failure probability. Thus, according to equations (1) and (2), the fuzzy probability of a system failure (top event occurrence) is determined from the fuzzy probabilities of components failure. The parameter a_i is the lower bound, the parameter m_i is the modal value, and the parameter b_i is the upper bound for each fuzzy probability of components failure. These parameters are given in Table 1.

Figure 7, provides the representation of computed fuzzy probability associated to the failure F, ds_1 and ds_2 . The fuzzy failure probability of the top event (F) is given below:

$$\text{Lower value } (a_i) = 0.002327,$$

$$\text{Middle value } (m_i) = 0.002911,$$

$$\text{Upper value } (b_i) = 0.004411.$$

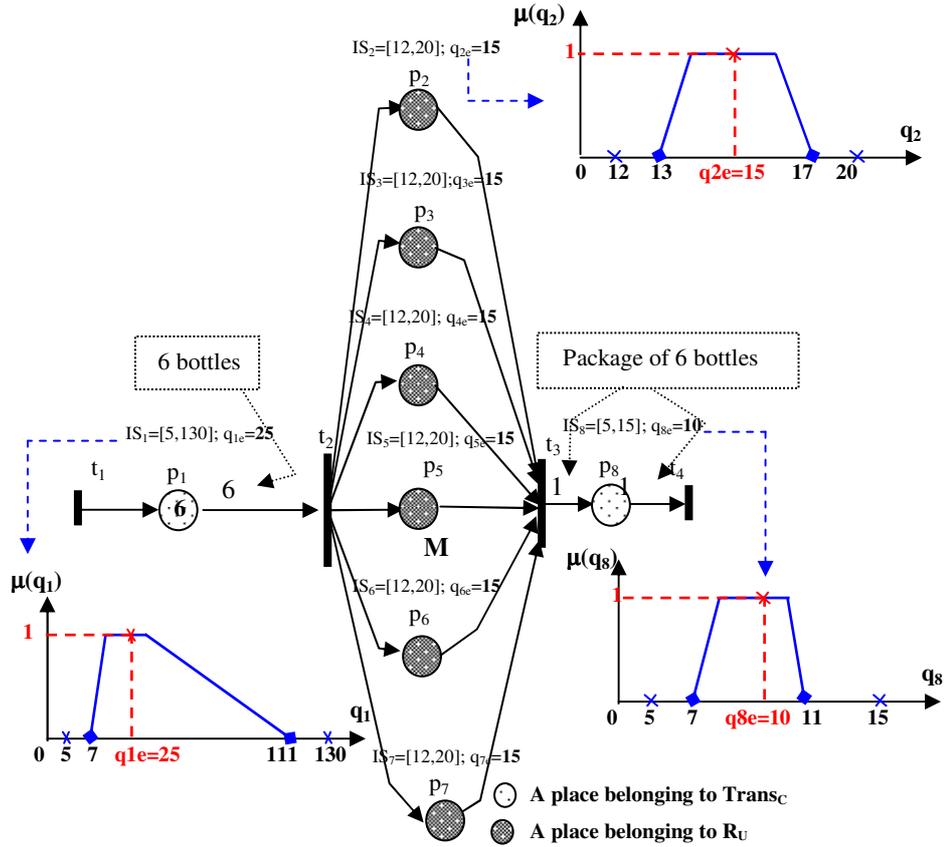


Fig.5. Packaging machine modeled by a P-time Petri net [5]

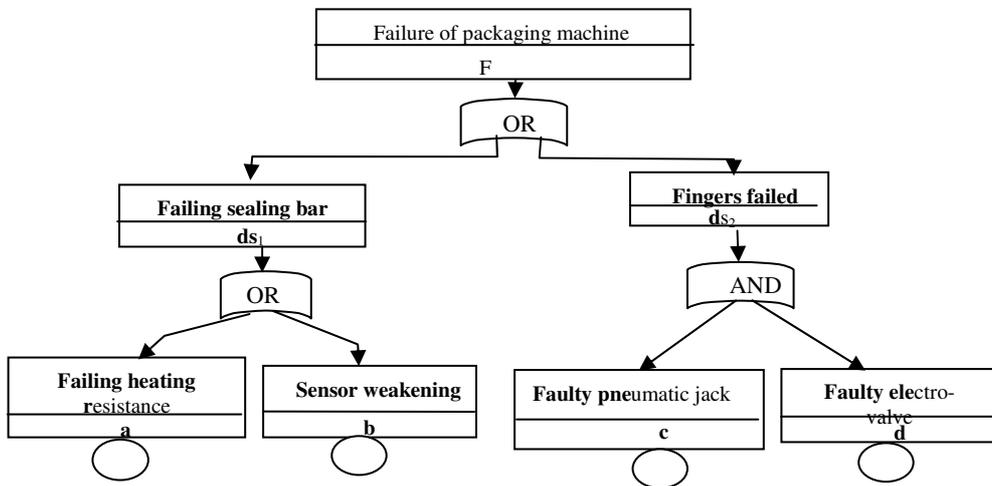


Fig.6. Fault Tree of packaging Machine

TABLE I. PARAMETERS OF FUZZY PROBABILITIES

Basic event	m	a	b
a	0,0015	0,00111	0,0023
b	0,0014	0,00121	0,0021
c	0,00413	0,0032	0,0048
d	0,0032	0,0025	0,0033

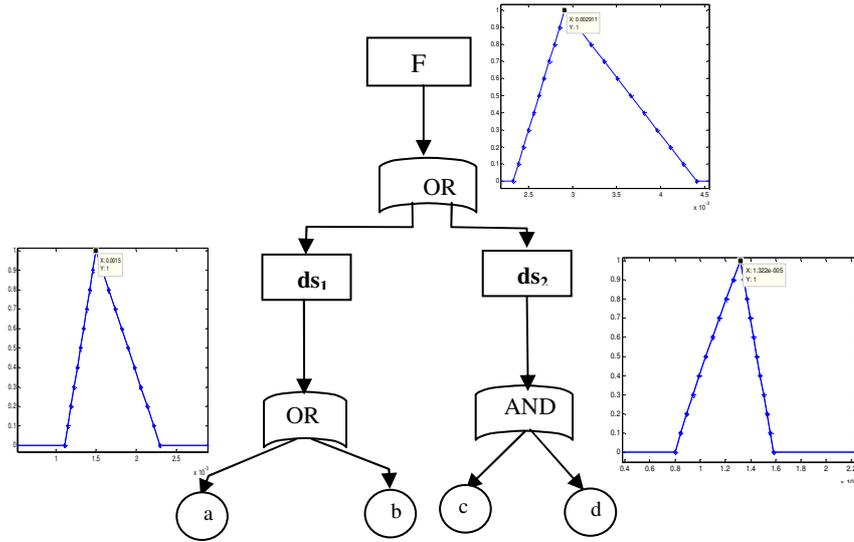


Fig.7. Fuzzy probability of a component failure (F, ds_1 and ds_2)

5.1.5 Maintenance Approach

To demonstrate the effectiveness and accuracy of the recovery approach, an example with three fuzzy rules is outlined. Consider the following fuzzy rules base:

- ✓ Rule 1: IF the sojourn time $q_2 \in [10, 12]$ AND the fuzzy failure probability of the top event “F” $Pr_F \in [0.0023, 0.0044]$ THEN there is a corrective maintenance.
 - ✓ Rule 2: IF the sojourn time $q_2 \in [13, 17]$ AND the fuzzy failure probability $Pr_F \in [0.0023, 0.0029]$ THEN there is a scheduled preventive maintenance.
 - ✓ Rule 3: IF the sojourn time $q_2 \in [17, 20]$ AND the fuzzy failure probability $Pr_F \in [0.0044, 0.1]$ THEN there is a corrective maintenance.
- Each rule use the operator "AND" in the premise, since it is an AND operation, the minimum criterion is used (Mamdani inference method), and the fuzzy outputs corresponding to these rules are represented by Figure 8.
 - Next we perform defuzzification to convert our fuzzy outputs to a single number (crisp output), various defuzzification methods were explored. The best one for this particular application: the centre of area (COA) defuzzifier. According to the COA method, the weighted strengths of each output member function are multiplied by their respective output membership function center points and summed. Finally, this area is divided by the sum of the weighted member function strengths and the result is taken as the crisp outputs. In practice there are two fuzzy outputs to defuzzify (corrective and preventive maintenance). Analysing the data, it is noted that the appropriate technique for recovery is the corrective.

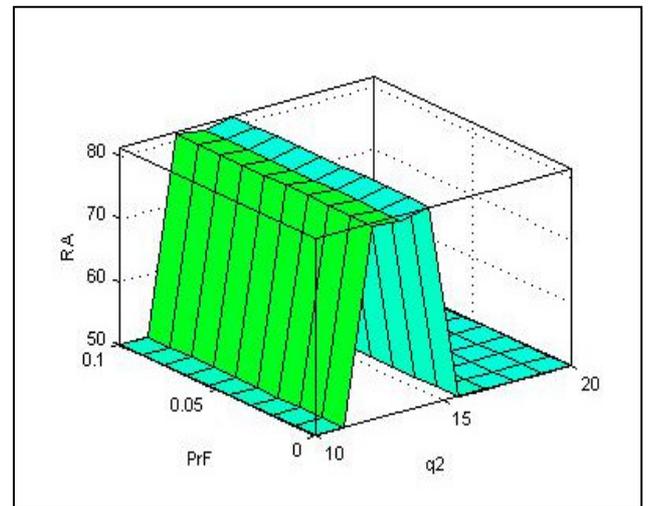


Fig.8. Three-dimensional trapezoidal membership function $[RA=f(q_2, Pr_F)]$

B. Processing station (Job Shop without Assembling Tasks)

1) Presentation of processing station

In the processing station, figure 9, workpieces are tested and processed on a rotary indexing table. The rotary indexing table is driven by a DC motor [19]. On the rotary indexing table, the workpieces are tested and drilled in two parallel processes. A solenoid actuator with an inductive sensor checks that the workpieces are inserted in the correct position. During drilling, the workpiece is clamped by a solenoid actuator. Finished workpieces are passed on via the electrical ejector, figure 9. The processing station is consists of [19]:

- ✓ Rotary indexing table module
- ✓ Testing module
- ✓ Drilling module

✓ Clamping module

2) Modeling of processing unit

Figure 10, shows a P-time Petri net (G) modeling the production unit. The obtained G is used to study the maintenance of processing unit. The full set time intervals of operations, in studied unit, are summarized in table II (u.t: unit time).

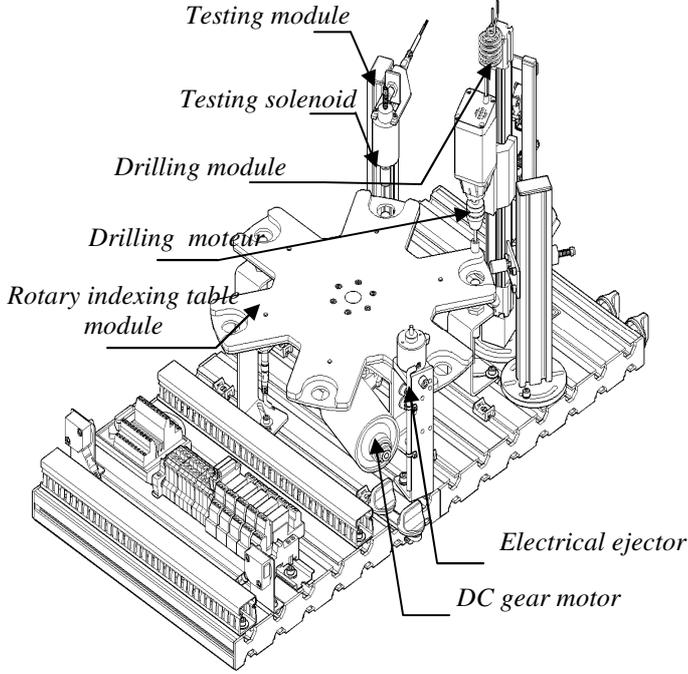


Fig. 9. Processing Station [19]

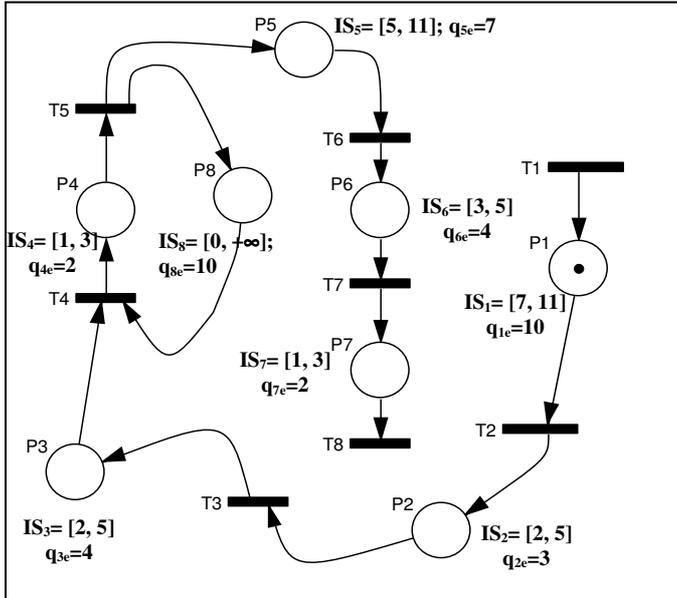


Fig. 10. Processing station modeled by a P-time Petri net

TABLE II. TIME INTERVALS ASSOCIATED TO OPERATIONS

Places	Action	IS _i (u.t)	q _{ie}
P1	Turn indexing table	[7, 11]	10
P2	Testing	[2, 5]	3
P3	Turn indexing table	[2, 5]	4
P4	Clamping	[1, 3]	2
P5	Drilling	[5, 11]	7
P6	Turn indexing table	[3, 5]	4
P7	Sorting	[1, 3]	3
P8	Retraction of clamping device	[0, +∞]	10

3) Monitoring of processing unit based on effective sojourn time

The purpose of the monitoring task is to detect, localise, and identify problems that occur on the system. These problems can be physical (a piece of equipment is down, a cable is cut) or logical (a station is rebooting, a logical connection is down...).

The considered approach uses the additional information provided by the knowledge of the effective sojourn time and allows detecting a failure symptom when a constraint is violated.

Let us suppose that we want to monitor the drilling platform. In this module, a clamping device clamps the workpiece. Once the drilling is completed, the drilling machine is stopped, moved to its upper stop and the clamping device is retracted, figure 9.

According to P-TPN, figure 10, the minimum time granted to the drilling operation is 5 u.t, whereas the maximum time is 11 u.t (IS₅ = [5, 11]; q_{5e} = 7).

Suppose that the drilling duration is 13 u.t (indicated by the effective sojourn time q₅ = 13 u.t and q₅ ∉ [a₅, b₅]). This delay of sojourn time, Figure 11, implies that:

- there is a technical failure of the production tool (clamping device, drilling machine, inductive sensor,...) requiring to generate a maintenance action,
- the quality of the manufactured product is incorrect since q₅ ∉ [a₅, b₅].

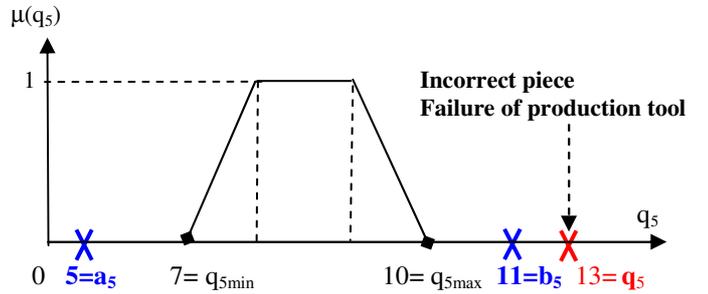


Fig.11. Function of possibility associated with an effective sojourn time (q₅)

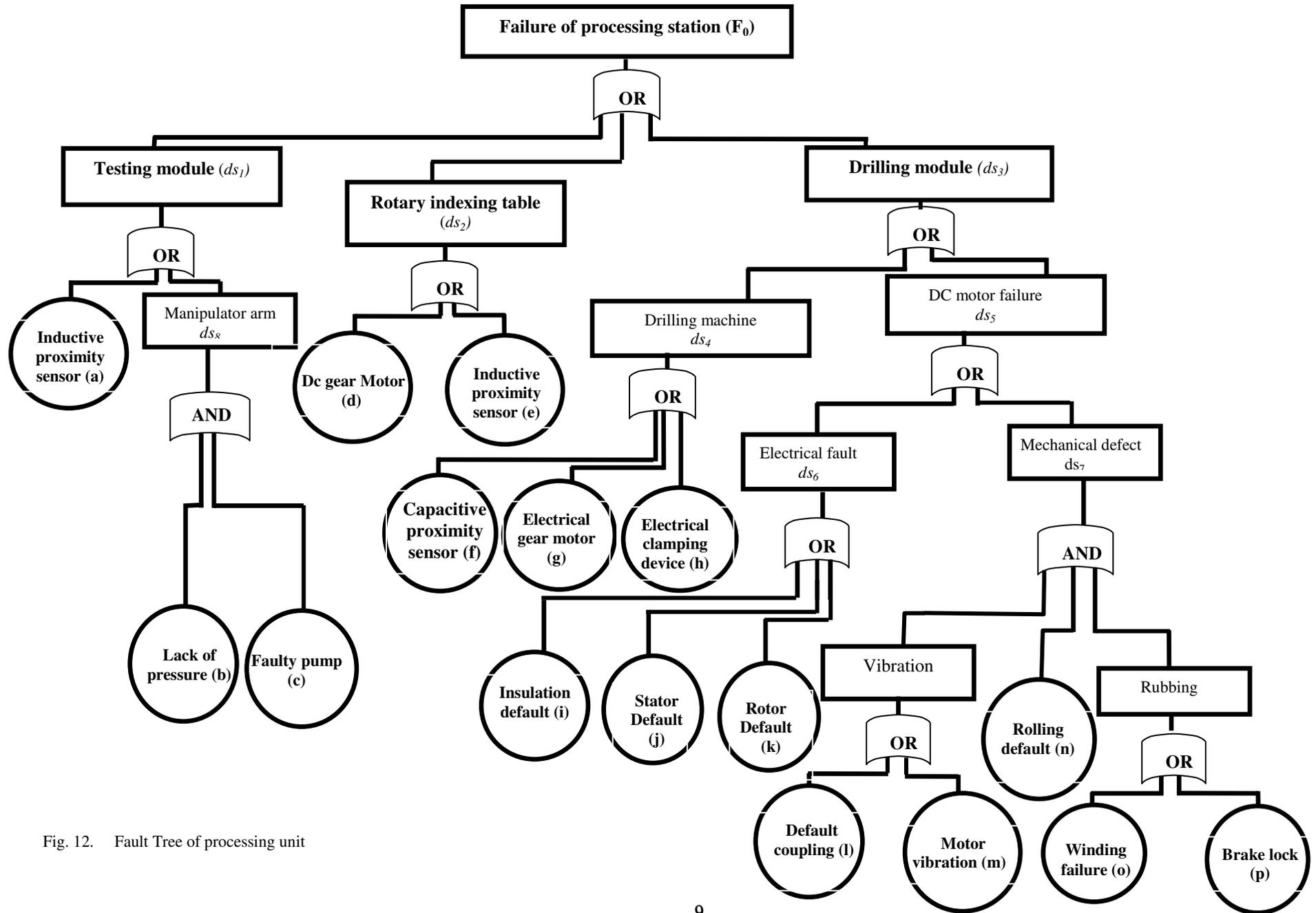


Fig. 12. Fault Tree of processing unit

4) Diagnosis of processing station

To establish the causality of failures on the sub-systems that can affect the system status, a fault tree, figure 12, was constructed and processing unit failure was defined as the top event of the fault tree (F_0). This diagnostic tree was comprised of 16 basic events.

The calculation of the probability allows us to determine the critical components of the tree and improve system reliability. In addition, this probability guide us in locating basic events that contribute to the vagueness of top event failure rates and thus effectively reduce this imprecision by a feedback on the vagueness of concerned elementary events.

The parameter a_i , m_i , b_i are given in Table III. We choose the trapezoidal shapes because of their mathematical simplicity. Figure 13, gives the fuzzy probability of the top event occurrence (P_{F0}). Analysing the data, it is noted that the most critical events in the fault tree are g, and i, respectively associated respectively to defaults d_g , d_i (greater probability value). Consequently we can deduce the most critical components to system failure; in fact a small variation in the critical component configuration causes a relatively greater change in the estimate of the top event failure probability.

TABLE III. PARAMETERS OF FUZZY PROBABILITIES

Basic event	m_1	m_2	a	b
d_a	0,0013	0,0018	0,00111	0,0023
d_b	0,0014	0,0014	0,00121	0,0021
d_c	0,0041	0,0041	0,0032	0,0048
d_d	0,0028	0,0032	0,0025	0,0033
d_e	0,0024	0,0028	0,0022	0,0038
d_f	0,0034	0,0034	0,0021	0,0043
d_g	0,009	0,009	0,0006	0,012
d_h	0,0012	0,0019	0,0009	0,0019
d_i	0,0015	0,018	0,0010	0,02
d_j	0,00191	0,00191	0,0017	0,00221
d_k	0,0053	0,0063	0,0045	0,0071
d_l	0,00505	0,00505	0,0041	0,0066
d_m	0,0053	0,0056	0,005	0,0061
d_n	0,00365	0,00365	0,0028	0,00676
d_o	0,0033	0,0035	0,0029	0,0042
d_p	0,00364	0,00364	0,0024	0,0044

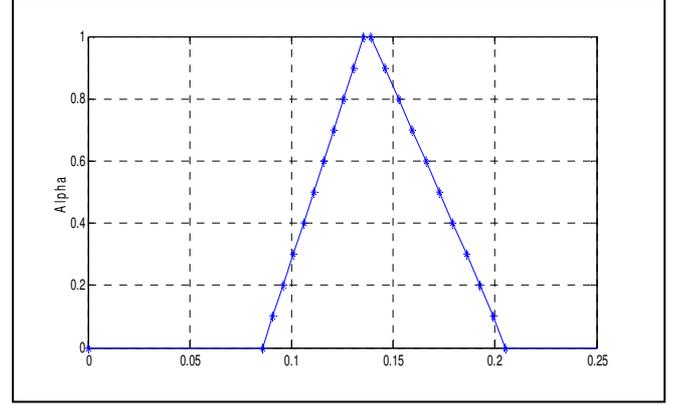


Fig. 13. Membership function for the top event failure probability

Based on probabilistic measures, the proposed maintenance model is able to evaluate the relative influence of components reliability on the reliability of the system and provide useful information about the maintenance strategy of these elements. The FPNM model is able to trigger one or more preventive or corrective actions.

Thus, the failures and repair process is capable of indicating when a failure (is about to) occur, so that repair can be performed before such failure causes damage or capital investment loss.

5) Maintenance of processing station

According to diagnosis information, the role of FPNM associated to the processing unit, figure 14, is to modify the control models, activate urgent procedures, finally, decide about the selective maintenance decision.

When the maintenance is triggered by the operator after an unresolved fault case – it is the corrective maintenance policy or when triggered by the statistic block - it is the preventive and predictive maintenance.

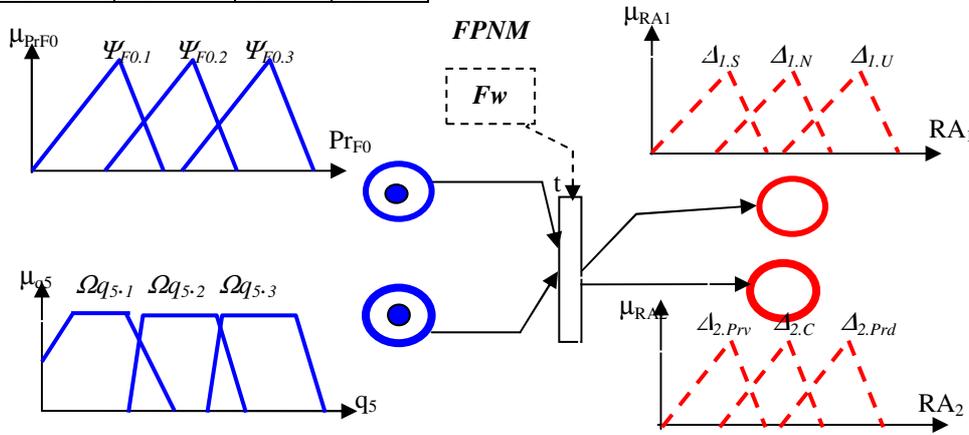


Fig. 14. FPNM of processing unit

The full set of linguistic variables associated to each input membership are summarised in table IV.

TABLE IV. LINGUISTIC VARIABLES ASSOCIATED TO THE INPUTS

Input	Membership	Linguistic variable
Pr _{F0}	$\Psi_{F0,1}$	Minor
	$\Psi_{F0,2}$	Average
	$\Psi_{F0,3}$	High
q ₅	$\Omega_{q5,1}$	Insignificant
	$\Omega_{q5,2}$	Marginal
	$\Omega_{q5,3}$	Critical

Similarly, table V shows linguistic variables associated to the output “recovery action”.

TABLE V. LINGUISTIC VARIABLES ASSOCIATED TO THE INPUTS

Output	Membership	Linguistic variable
RA ₁	$\Delta_{1,S}$	Slow
	$\Delta_{1,N}$	Normal
	$\Delta_{1,U}$	Urgent
RA ₂	$\Delta_{2,Prv}$	Preventive
	$\Delta_{2,C}$	Corrective
	$\Delta_{2,Prd}$	Predictive

It is necessary to point out the purpose of the FPNM. As soon as this block is requested by the diagnosis block, it triggers different actions (slow, normal or urgent), figure 14. If there is a risk to the operator or the process, the proposed FPNM triggers an emergency procedure.

If the expected state is not coherent with the reference state (operations that are not in conformity with the process state), the system sets to maintenance state: This is the corrective maintenance scenario. In the case of corrective maintenance, it is necessary to “repair the defective material”, eliminate fault effects in order to reach the system’s regular operation status.

When maintenance is required (corrective, preventive or predictive), the FPNM model inhibits all pre-set regular operating conditions at the modelling and diagnosis level. At this point the maintenance module takes up control of the process. The maintenance task is, therefore, synchronized with the modelling and diagnosis model.

VI. CONCLUSION

In this paper, we have proposed a fuzzy Petri net for maintenance, able to analyze monitoring and recovery tasks of manufacturing systems with time constraints. The new recovery approach is based on the study of effective sojourn time of the token in places and the evaluation of the failure probability of fault tree events.

Our study makes the assumption that the supervised system is modeled by P-time Petri nets. The paper proposes an adaptive technique dedicated to the maintenance of

manufacturing systems with time constraints. This model has a double interface, one with the modeling model system and the second one with the behavioral model (diagnosis)

At the occurrence of a dysfunction in a milk packaging machine, it is important to react in real time to maintain the productivity and to ensure the safety of the system. It has been shown that the knowledge of the effective sojourn time of the token has a significant contribution regarding this type of problem, since it makes the supervision more efficient by an early detecting of a time constraint violation. This is quite useful for the maintenance task.

We have developed and used a fuzzy probabilistic approach to evaluate the failure probability of the top event, when there is an uncertainty about the components failure probabilities. This approach is based on the use of fuzzy probabilities.

To illustrate the efficiency of the maintenance approach, we have applied it to a packaging process. The proposed Petri net approach can achieve early failure detection and isolation for fault diagnosis. These capabilities can be very useful for health monitoring and preventive maintenance of a system.

Based on two workshops topology, it can be claimed that the proposed fuzzy Petri nets for maintenance allows applying various maintenance policies -corrective, preventive and predictive.

It is interesting as further research to incorporate the issues of maintenance and repair strategies into the fuzzy probabilistic approach in order to compute a modified maintenance cost. This last problem needs a specific approach, because of the production loss which occurs when maximum time constraints are not fulfilled anymore.

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