"International Journal of Control, Energy and Electrical Engineering (CEEE)

Vol.1, pp.75-79

LMI Approach to Stabilization of Descriptor Systems Via Proportional Plus Derivative State Feedback

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***Abstract*— This paper is concerned with a proportional plus derivative (PD) state feedback control problem for discrete and continuous descriptor linear systems. We aim to design a (PD) state feedback controller which guarantees the stability of closed-loop system.**
The existence of such a controller is determined by developing a necessary and sufficient condition in terms of LMIs. Then, the desired PD state feed-back controller is given in the explicit expression. The proposed approaches' applicability is illustrated by an example of simulation.

***Keywords*—descriptor linear systems; proportional plus derivative state feedback; linear matrix inequality**.

I. Introduction

The Singular systems, also called descriptor system, implicite systems or algebro differentiel systems present an important class of systems with a great practical and theoretical interest. This class was firstly used for the modelization of a large range of systems that can not be modelized by the usual state representation. Indeed, descriptor systems chow both dynamic relations and algebraic ones. This augmentation allows adding static relations in the modelization of process that have an impulsive behavior or also non causal process. Besides, descriptor systems keep the systems physical significations [1]. [1, 2] Singular systems are used in electric, chemical and robotic fields. Since 1970, many researches were concentrated on descriptor systems. A several number of fundamental results obtained for ordinary systems, have been extended for singular systems such that: observability, controllability stability, elimination of impulse behavior, pole assignment, [2-7]. A great interest in the area of stability, stabilization techniques and robustness for descriptor systems has been noted [8-12]. Interested readers may refer to [13], where a comparison between, the concept of Lyapunov functions and the theory of differential inequalities is established for singular system. In the work of [14], a problem of regularization by state and output predictive controllers is treated firstly. Then a stabilization procedure of the regularized system is given and a computation of controller gains through linear matrix inequalities is developed. In [15], a proposed approach based on GLE is adopted under a set of matrix inequality for the admissibility of discrete singular systems. This last property includes the stability as well as the impulse freeness and the regularity. Other works have been interested in the robust stabilization [16, 17]. [18] has developed the robust stability of singular delayed systems by introducing the concept of generalized quadratic stability. A strict LMI design approach is proposed and an explicit expression for robust state feedback control law is given. Moreover, in [19], the robust stabilization problem is solved through state feedback controller where the parameters uncertainties appearing in both the state and input matrices and the concepts of generalized quadratic stability and stabilizabilty are introduced. . In [20], to reduce the conservatism of the stabilization quadratic, a PDL approach is used to solve the robust static output-feedback admissibility problem, for the descriptor systems case. The aim of this work is to study a proportional plus derivative (PD) state feedback controller for a nominal continuous descriptor system, satisfying the closed-loop systems stability. Based on this result, a necessary and sufficient condition for the solvability of this problem is obtained in terms of linear matrix inequality LMIs.

This present study has a different aspect from those were developed in literature for singular systems stabilization. The different consists on the controller structure, since it is the first work that dealing with the PD regulator case and on the stabilization technique. This latter exploits the stabilization concept for the standard usual systems. The point is to determine a PD controller that translating the study from a descriptor system to a standard one. Then, we look for a necessary and sufficient condition under a LMI formalism witch guarantying the system stabilization. The paper is organized as following: The second section formulates the problem to be addressed in the paper. Some necessary and sufficient conditions for stabilization of descriptor linear systems via state plus state derivative feed-back are presented in section three where the both continuous and discrete cases will be treated. An illustrative example is worked in section four. The last section gives some concluding remarks.

***Notations**:* Throughout this paper, the following notations will be used. For two matrices A and B, A>B means that A-B is positive definite. A^T denotes the transpose of A and A^{-T} the transpose of the inverse of A. Identity and null matrices will be denoted respectively by I and 0.

Furthermore, in the case of portioned symmetric matrices, the symbol $*$ denotes generally each of its symmetric blocks and $A+sym(*)$ denotes $A+A^T$.

II. PRELIMINARIES

Let's consider the following continuous-time linear descriptor system described by:

$$E\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

Where $x(t)\in\mathbb{R}^n$, $u(t)\in\mathbb{R}^m$ and $y(t)\in\mathbb{R}^r$ are its state, control input vector, and measurement output respectively. $E, A\in\mathbb{R}^{n\times n}$ $B\in\mathbb{R}^{n\times m}$, and $C\in\mathbb{R}^{r\times n}$ are constant matrices of appropriate dimensions. The matrix E may be singular. It is assumed that $rank(E) = q \le n$.

III. STABILIZATION PROBLEM

A. Stabilization condition for continuous –time descriptor systems.

We consider the state proportional plus derivative control expressed as:

$$u(t) = K_p x(t) - K_d \dot{x}(t) \tag{2}$$

Where $K_p, K_d \in\mathbb{R}^{m\times n}$ are matrixes of appropriate dimensions. The closed loop system is given by:

$$(E+BK_d)\dot{x}(t) = (A+BK_p)x(t) \tag{3}$$

We aim to design a PD state feed-back control of the form (2) such the gain K_p acts on the stability of the system and K_d make the expression (3) well defined. In other words find a gain K_d, which allows us to write the equation (3) as follows:

$$\dot{x}(t) = (E+BK_d)^{-1}(A+BK_p)x(t) \tag{4}$$

Thus, the stabilization study of system (1) becomes a study of usual standard system. So we shall avoid studying the concepts of singular systems such as the regularity, the impulsiveness behavior and the admissibility.

Let us introduce the following definition, where the stability is stressed under the Lyapunov sense.

***Definition*1:** System (1) is SD-stabilizable if there exist matrix K_p and K_d of appropriate dimensions and positive definite symmetric P such that:

$$(E+BK_d)^{-1}(A+BK_p)P + P(A+BK_p)^T(E+BK_d)^{-T} < 0 \tag{5}$$

This definition supposes that system (4) is well defined, i.e that matrix $(E+BK_d)$ is full rank. Consequently matrix $(A+BK_p)$ is also full rank.

1. *Stabilization for a Nominal Descriptor System*

In this section, a necessary and sufficient condition for the solvability of PD state feedback controller is given in terms of linear matrix inequalities. The theorem below follows directly from the previous definition.

***Theorem*1:** the following statements are equivalent:

i) System (1) is SD-stabilizable.

ii) There exist a positive definite matrix Y and matrices F, R_1 and R_2 of appropriate dimensions such that:

$$\begin{bmatrix} F^TA^T + AF + BR_1 + R_1^TB^T & Y + F^TA^T + R_1^TB^T - EF - BR_2 \\ * & -F^TE^T - EF - BR_2 - R_2^TB^T \end{bmatrix} < 0 \tag{6}$$

The gain given by: $K_p = R_1F^{-1}$ and $K_d = R_2F^{-1}$ solves problem.

Proof: by the definition1, the system (1) is SD-stabilizable by PD state feedback (2) if and only if the equation (5) checked. By taking $Q = P^{-1}$, Q>0, and applying the projection lemma [3], it exist a matrix G of appropriate dimension such that:

$$\begin{bmatrix} (A+BK_p)^TG^T + G(A+BK_p) & (A+BK_p)^TG^T + Q - G(E+BK_d) \\ * & -(E+BK_d)^TG^T - G(E+BK_d) \end{bmatrix} < 0$$

Taking now $F = G^{-T}$, and applying the congruence transformation $diag(G^{-1}, G^{-1})$ and denoting:

$Y = G^{-1}QG^{-T}$, $R_1 = K_pF$ and $R_2 = K_dF$, one gets the inequality (6). This completes the proof of theorem.

2. *Extension to Robust Stabilization*

The result of the previous section can be extended to robust stabilization for uncertain descriptor systems with polytopic coefficient matrices.

Lets us consider the following linear uncertain descriptor system:

$$E(\theta)\dot{x}(t) = A(\beta)x(t) + B(\zeta)u(t) \tag{7}$$

Where $x \in \mathbb{R}^n$ is the state vector, we assume that *E*, *A* and *B* are constant and respectively belong to the classes:

$$E \in \mathcal{E} = \left\{ E(\theta) : E(\theta) = \sum_{i=1}^{N_E} \theta_i E_i, \sum_{i=1}^{N_E} \alpha_i = 1, \alpha_i \geq 0 \right\}$$

$$A \in \mathcal{A} = \left\{ \mathrm{A}(\beta) : A(\beta) = \sum_{j=1}^{N_A} \beta_j A_j, \sum_{j=1}^{N_A} \beta_j = 1, \beta_j \geq 0 \right\}$$

$$B \in \mathcal{B} = \left\{ B(\zeta) : B(\zeta) = \sum_{k=1}^{N_k} \zeta_k B_k, \sum_{k=1}^{N_k} \zeta_k = 1, \zeta_k \geq 0 \right\}$$

***Theorem2*:** the following statements are equivalent:

i) System (7) is SD-stabilizable.

ii) There exist a positive definite matrix *Y* and matrices *F*, R_1 and R_2 of appropriate dimensions such that:

$$\begin{bmatrix} F^T A_i^T + A_i F + B_j R_1 + R_1^T B_j^T & Y_{i,j,k} + F^T A_i^T + R_1^T B_j^T - E_k F - B_j R_2 \\ * & -F^T E_k^T - E_k F - B_j R_2 - R_2^T B_j^T \end{bmatrix} < 0 \quad (8)$$

$$i = 1 \dots N_E, \; j = 1 \dots N_A, \; k = 1 \dots N_B$$

The gain given by: $K_p = R_1 F^{-1}$ and $K_d = R_2 F^{-1}$ solves problem.

Proof: The proof follows by simple convexity arguments. Notes that $\sum_{i=1}^{N_A} \sum_{j=1}^{N_B} \sum_{k=1}^{N_E} \beta_i \theta_j \zeta_k Y_{i,j,k}$ is a Lyapunov function for the closed-loop system.

Remark1: compared with the result obtained in [9], for the robust stabilization, the necessary and sufficient condition proposed for the determination of the (PD) controller structure, is just limited for the proportional gain (K_2). However, the derivative gain must be choosing from the beginning for the resolution of the problem. On the other sides, the present contribution is different for the reason that the both constants K_p and K_d will be induced by LMI solvers, which highlights the interest of theorem 2.

B. Stabilization condition for discrete-time descriptor systems.

In this subsection, the methodology can be extended for the discrete-time.

Let a given discrete-time descriptor system be:

$$Ex(k+1) = Ax(k) + Bu(k) \qquad (9)$$

Where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$ and $y(k) \in \mathbb{R}^r$ are its state, control input vector, and measurement output respectively. $E, A \in \mathbb{R}^{n \times n}$ $B \in \mathbb{R}^{n \times m}$, and $C \in \mathbb{R}^{r \times n}$ are constant matrices of appropriate dimensions. The matrix *E* may be singular. It is assumed that $rank(E) = q \leq n$.

Similar to the results obtained in the previous section, the (PD) state feedback law: $u(k) = K_p x(k) - K_d x(k+1)$ ensures the stability of the closed loop system:

$$(E + BK_d)x(k+1) = (A + BK_p)x(k) \qquad (10)$$

where the feedback gain K_d guarantees the invertibility of the expression $(E + BK_d)$.

So, if $(E + BK_d)$ is non singular, (10) can be rewritten as:

$$x(k+1) = (E + BK_d)^{-1}(A + BK_p)x(k)$$

Before giving the result of this subsection, we would expose the preliminary definition.

***Definition2*:** System (9) is SD-stabilizable if there exist matrix K_p and K_d of appropriate dimensions and positive definite symmetric *P* such that:

$$((E + BK_d)^{-1}(A + BK_p))^T P (E + BK_d)^{-1}(A + BK_p) - P < 0 \qquad (11)$$

The solvability of the stabilization problem of the system (9) is given by the following theorem.

***Theorem3*:** the following statements are equivalent:

i) System (9) is SD-stabilizable.

ii) There exist positive definite matrixes *P*, matrices *G*, R_1 and R_2 of appropriate dimensions and scalar $\alpha > 0$, sufficiently large, such that:

$$\begin{bmatrix} P + \alpha G^T E^T + \alpha EG + \alpha BR_2 + \alpha R_2^T B^T & -\alpha G^T A^T - \alpha R_1^T B^T \\ * & -P \end{bmatrix} < 0 \quad (12)$$

The gain given by: $K_p = R_1 G^{-1}$ and $K_d = R_2 G^{-1}$ solves problem.

Proof: According to Lyapunov stability theory (definition2), the closed loop system (10) is SD – stabilizable if and only if:

$$E_c^{-T} P E_c^{-1} - A_c^{-T} P A_c^{-1} < 0$$

where $A_c = A + BK_p$ and $E_c = E + BK_d$

Applying the projection Lemma [3], it exist a matrix *G* of appropriate dimension such that:

$$\begin{bmatrix} P + G^T E^T + EG + BK_d G + G^T K_d^T B^T & -G^T A^T - G^T K_p^T B^T \\ * & -P \end{bmatrix} < 0$$

Taking now $R_1 = K_p F$ and $R_2 = K_d F$, the inequality (12) follows.

IV. NUMERICAL ILLUSTRATION

To demonstrate the effectiveness and applicability of the proposed method of the stabilization, we provide the following three examples, the first one concerned a continuous nominal system, the second introduced the uncertainties and the last concerned a discrete nominal descriptor system.

A. Example-1

We consider a nominal linear descriptor system with parameters as follows:

$$E=\begin{bmatrix}0&0&0\\1&0&0\\0&1&0\end{bmatrix},\ A=\begin{bmatrix}-1&4.5&-0.5\\-7&7&-8\\-5&3&-6\end{bmatrix},\ B=\begin{bmatrix}1&0&1\\0&1&1\\1&0&-1\end{bmatrix}.$$

We use the MATLAB Control Toolbox to solve the LMI in (6), we obtain the parameters of controller as follows:

$$K_p=\begin{bmatrix}-85.5927&7.2735&-144.9593\\0.7641&10.4851&9.4792\\-15.6521&56.5848&1.3652\end{bmatrix},K_d=\begin{bmatrix}87.6802&-8.9111&147.8536\\-4.4103&-3.1549&-11.6439\\19.1912&-61.0209&3.1189\end{bmatrix}$$

The following figures illustrate the time behavior of states of the nominal descriptor system, with an initial value of the state: $x(t)=\begin{bmatrix}1&0&-1\end{bmatrix}^T$. This justifies the effectiveness of proposed approach considering the stability criteria.

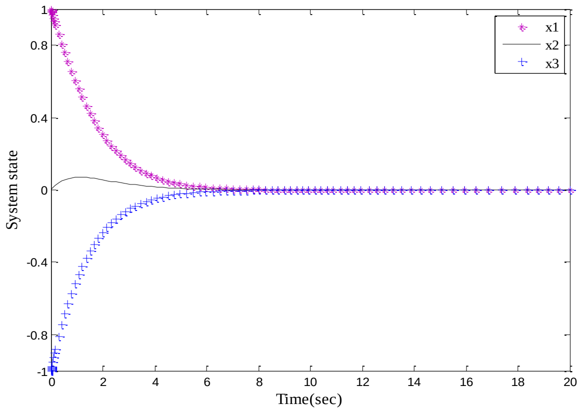

Fig.1 System state trajectories.

B. *Example-2*

We consider the vertices of the polytopic, are given by triple:

$$(E_k,A_i,B_j)=\begin{Bmatrix}(E_1,A_1,B_1),(E_1,A_1,B_2),(E_1,A_2,B_1),(E_1,A_2,B_2)\\(E_2,A_1,B_1),(E_2,A_1,B_2),(E_2,A_2,B_2),(E_2,A_2,B_1)\end{Bmatrix}$$

where:

$$E_1=\begin{bmatrix}0&0&2&0\\0&1&0&0\\-1&0&0.8&0\\0&0&0&0\end{bmatrix},E_2=\begin{bmatrix}0&0&2&0\\0&1&0&0\\-1&0&0.8&0\\0&0&0&0\end{bmatrix}$$

$$A_1=\begin{bmatrix}5.4&0&0&0\\0&4&0&0\\-1&0&3&0\\0&1&0&-2\end{bmatrix},A_2=\begin{bmatrix}6.4&0&0&0\\0&4&0&0\\-1&0&3&0\\0&1&0&-2\end{bmatrix}$$

$$B_1=\begin{bmatrix}1&0\\0&1\\1&0\\0&1\end{bmatrix},B_2=\begin{bmatrix}1&0\\0&1\\1&1\\0&1\end{bmatrix}$$

The resolution of this example shows a feasible solution was the following:

$$K_d=\begin{bmatrix}-0.5382&-0.2097&-1.6410&0.1193\\0.5539&0.1726&0.5413&-0.3149\end{bmatrix},K_p=\begin{bmatrix}8.9776&-2.2254&2.4211&0.6274\\-1.6532&9.73013&2.5013&-3.2269\end{bmatrix}$$

B. *Example-3*

Consider the discrete singular system (9) with

$$E=\begin{bmatrix}1&1&1\\1&1&1\\0&0&1\end{bmatrix},\ A=\begin{bmatrix}0.1&-0.3&2\\0.5&-3&-1\\0.2&0.4&-0.1\end{bmatrix},B=\begin{bmatrix}-1&1\\1&-1\\1&1\end{bmatrix}$$

Using in LMI solver, theorem (3) is applied with $\alpha=20$. One obtains the (PD) state feedback matrix:

$$K_d=\begin{bmatrix}59.4500&-272.3781&93.3153\\-52.0309&237.2821&-82.4852\end{bmatrix},$$

$$K_p=\begin{bmatrix}-3.1448&13.9595&-3.8551\\2.6349&-12.8823&3.4415\end{bmatrix}$$

We can see in the figure2, the time behavior of states, with an initial value of the state: $x(t)=\begin{bmatrix}1&0&-1\end{bmatrix}^T$, that by using our controller synthesis procedure, that the trajectory of the closed loop system is stable.

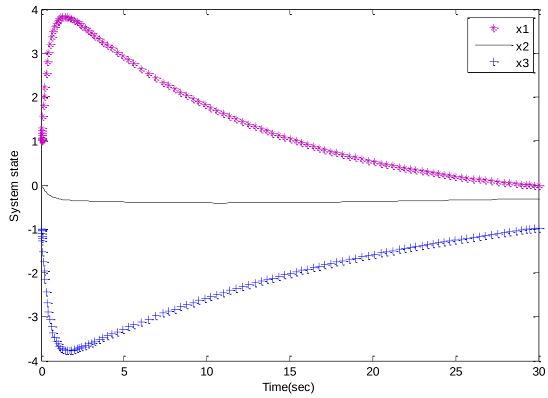

Fig.2 System state trajectories

V.CONCLUSION

The present paper provided a necessary and sufficient condition for the existence of proportional plus derivative feedback controllers for descriptor systems. It represented an LMI based approach to the design of PD state feedback controller. This result is extended for a robust stabilization problem. Numerical examples have shown the effectiveness of the proposed approaches considering the stability criteria.

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