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Construction of Lyapunov function to examine Robust Stability for Linear System

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***Abstract* —In the recent years, there is a growing interest to the robust stability approach for representation and control of linear and nonlinear complex systems. This paper detailed describes a new approach to ensure robustness for research linear system. The robust stability of control systems where the controlled plant possesses dynamics is relevant today. The originality of this paper focuses on robust stability analysis of system with design of Lyapunov function and defines conditions. We describe a method for construct the Lyapunov function for linear system via applying geometric interpretation. Finally we made comparative analysis of examples and for all the examples the stability conditions of the system executed. The results proved that the area of robust stability can be extended using the proposed approach. This work presents some theoretical fundamental and practical results and general problem of robust stability is defined.**

***Keywords* — Stability, Linear systems, Robust control, Control theory, Lyapunov function.**

I. INTRODUCTION

Contemporary control approach is characterized by increased dynamics requiring stability and quality control of processes conditioned by multiple restrictions and incomplete data.

Key task is to investigate the problem of behavioral change of control systems at ultimate parameters changes and control law synthesis that provides for optimal protection from indefiniteness in research of object properties.

The robustness is assumed as an ability to maintain system availability in a condition of parametric and non-parametric indeterminateness [1,2] describing control objects. The most important idea in the study of robust stability is to specify constraints for changes in control system parameters that preserve stability.

Studies [3,4] etc dedicated to study robust stability of control systems. Nevertheless, many of them focused on studies of robust stability of linear continuous and discrete control systems specifically of characteristic polynomial, frequency characteristics and Lyapunov matrix equation.

For the purpose of studying the system dynamics and their control, we considered models based on observations of input and output signals of the object and the representing its behavior in the state space as most suitable.

There is provided a method for building Lyapunov function based on geometric interpretation of Lyapunov's direct (second) method [5] on gradient of dynamic systems in relation to some potential function in the state space of dynamic system.

II. MATHEMATICAL SYSTEM

The control system is given by the linear equation.

$$\dot{x} = Ax + Bu, x \in R^n, u \in R^m \quad (1)$$
$$y = Cx, y \in R^l$$

The controller is described by the equation

$$u = -Kx \quad (2)$$

or $$u_i = -k_{i1}x_1 - k_{i2}x_2 - ... - k_{in}x_n, i = 1,2,...,m$$

Description of parameters

$$A \in R^{n\times n},\ B \in R^{n\times m},\ C \in R^{l\times n}, K \in R^{m\times n}$$

Matrices of the object, control, output and coefficients of control system, $x(t) \in R^n$ - state vector, $u(t) \in R^m$ - vector control, $y(t) \in R^l$ - vector output of the system.

We can provide equation (1) in expanded form:

$$\begin{aligned}
\dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n + b_{11}u_1 + b_{12}u_2 + ... + b_{1m}u_m \\
\dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n + b_{21}u_1 + b_{22}u_2 + ... + b_{2m}u_m \\
&... \qquad\qquad ... \\
\dot{x}_n &= a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n + b_{n1}u_1 + b_{n2}u_2 + ... + b_{nm}u_m
\end{aligned} \quad (3)$$

Let us denote $G = A - BK$ matrix of the closed system and the system (3) in matrix-vector form, we can write

$$\dot{x} = G\,x,\ x(t) \in R^n,$$
$$g_{ij} = a_{ij} - \sum_{k=1}^{m} b_{ik}k_{kj}$$

Therefore equation (3) can be written as

$$\begin{cases} \dot{x}_1 = \left(a_{11} - \sum_{k=1}^{m} b_{1k}k_{k1}\right)x_1 + \left(a_{12} - \sum_{k=1}^{m} b_{1k}k_{k2}\right)x_2 + \ldots + \left(a_{1n} - \sum_{k=1}^{m} b_{1k}k_{kn}\right)x_n \\ \dot{x}_2 = \left(a_{21} - \sum_{k=1}^{m} b_{2k}k_{k1}\right)x_1 + \left(a_{22} - \sum_{k=1}^{m} b_{2k}k_{k2}\right)x_2 + \ldots + \left(a_{2n} - \sum_{k=1}^{m} b_{2k}k_{kn}\right)x_n \\ \ldots \\ \dot{x}_n = \left(a_{n1} - \sum_{k=1}^{m} b_{nk}k_{k1}\right)x_1 + \left(a_{n2} - \sum_{k=1}^{m} b_{nk}k_{k2}\right)x_2 + \ldots + \left(a_{nn} - \sum_{k=1}^{m} b_{nk}k_{kn}\right)x_n \end{cases} \quad (4)$$

III. GEOMETRICAL INTERPRETATION OF LYAPUNOV FUNCTION

Certain functions or so-called Lyapunov functions are used as basic instruments in Lyapunov direct method and based on two Lyapunov theorems. Lyapunov theorems have simple geometric interpretation. This interpretation not only determine main contents of the theorem but also could be used to solve problems in building Lyapunov function.

The direct method is a great advantage in the case of nonlinear systems. The method of constructing a Lyapunov function for stability determination is called the second method of Lyapunov. We use the «second method of Lyapunov» or the «direct method» as applied to linear systems.

The geometric meaning of a Lyapunov function used for determining the system stability around the zero equilibrium and it is shown schematically in Figures 1.

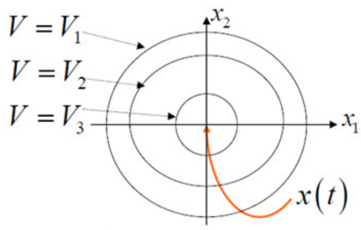

Fig. 1. Lyapunov function

The geometric identification of stable states is reduced to creating a family of closed surfaces that surround the zero equilibrium of coordinates. The system state moves across contour curves: each integrated curve can cross each of these surfaces.

We suppose that there exists a positive definite function $V(x_1, x_2, ..., x_n)$ for which $(dV/dt < 0)$, and consider any integral curve of (3), coming out at the initial time of any point of the origin.

If dV/dt is a function with negative definite $(dV/dt < 0)$, then every integral curve starting from a sufficiently small neighborhood of the origin, will be sure to cross each of the surfaces $V(x_1(t), x_2(t), .., x_n(t)) = C, C = const$ of the outside to the inside, as the $V(x_1(t), x_2(t), .., x_n(t)) = C$ function is continuously decreasing.

But in this case the integral curves have to be infinitely close to the origin, i.e. unperturbed motion is asymptotically stable [2].

Thus, from the geometric interpretation point of view the second method of Lyapunov, the study of stability is reduced to the construction of a family of closed surfaces surrounding the origin. As the integral curves have property to intersect each of these surfaces, then stability of the unperturbed motion will be set [2].

Let us consider, that the expression $dV(x)/dt < 0$ means, that

$$\frac{dV(x)}{dt} = \frac{\partial V(x)}{\partial x}\frac{dx}{dt} = |grad\, V(x)|\left|\frac{dx}{dt}\right| \cos\alpha < 0,$$

i.e. scalar product of the gradient vector Lyapunov functions $grad\, V(x)$ by the velocity vector dx/dt for the asymptotic stability of the system must be less than zero.

This condition will be true if the angle α between the gradient of the Lyapunov function $grad\, V(x)$ and the velocity vector dx/dt forms an obtuse angle $90^0 < \alpha \le 180^0$.

The gradient vector of the Lyapunov function is always directed from the origin toward the highest growth of Lyapunov functions.

Also note that, in the study of stability [1] the origin corresponds to the stationary states of the system or the set of the system. The state equation (1) or (4) shall be made in respect to deviations from the steady state $X_s\,(x = \Delta x = X(t) - X_s(t))$.

Therefore the left side of (1) or (4), dx/dt expresses the velocity vector changes and deviations. We can assume that the velocity vector of deviations submitted to the stability of a system to the origin.

Components of the gradient vector Lyapunov functions in the opposite direction, but they are equal in absolute value. Then, if the Lyapunov function $V(x)$ is specified as a vector of functions $V(V_1(x), V_2(x), ..., V_n(x))$ then gradient vector Lyapunov function can be written as $\partial V/\partial x = -dx/dt = -(A - BK)x$.

Vector components of the gradient of a potential function $V(x_1, ..., x_n)$ are given in the form of vector Lyapunov functions with components $(V_1(x_1, x_2, ..., x_n), V_2(x_1, x_2, ..., x_n), ..., V_n(x_1, x_2, ..., x_n))$ we write in the form:

$$\begin{cases} -\dfrac{dx_1}{dt} = \dfrac{\partial V_1(x)}{\partial x_1} + \dfrac{\partial V_1(x)}{\partial x_2} + ... + \dfrac{\partial V_1(x)}{\partial x_n} \\ -\dfrac{dx_2}{dt} = \dfrac{\partial V_2(x)}{\partial x_1} + \dfrac{\partial V_2(x)}{\partial x_2} + ... + \dfrac{\partial V_2(x)}{\partial x_n} \\ .... \\ -\dfrac{dx_n}{dt} = \dfrac{\partial V_n(x)}{\partial x_1} + \dfrac{\partial V_n(x)}{\partial x_2} + ... + \dfrac{\partial V_n(x)}{\partial x_n} \end{cases} \quad (5)$$

In this system by substituting values of the components of the velocity vector we get:

$$\begin{cases} \dfrac{\partial V_1(x)}{\partial x_1} + \dfrac{\partial V_1(x)}{\partial x_2} +,...,+ \dfrac{\partial V_1(x)}{\partial x_n} = \\ -\left(a_{11} - \sum_{k=1}^{m} b_{1k}k_{k1}\right)x_1 - \left(a_{12} - \sum_{k=1}^{m} b_{1k}k_{k2}\right)x_2 -,...,-\left(a_{1n} - \sum_{k=1}^{m} b_{1k}k_{kn}\right)x_n, \\ \dfrac{\partial V_2(x)}{\partial x_1} + \dfrac{\partial V_2(x)}{\partial x_2} +,...,+ \dfrac{\partial V_2(x)}{\partial x_n} = \\ -\left(a_{21} - \sum_{k=1}^{m} b_{2k}k_{k1}\right)x_1 - \left(a_{22} - \sum_{k=1}^{m} b_{2k}k_{k2}\right)x_2 -,...,-\left(a_{2n} - \sum_{k=1}^{m} b_{2k}k_{kn}\right)x_n \\ ... \\ ... \\ \dfrac{\partial V_n(x)}{\partial x_1} + \dfrac{\partial V_n(x)}{\partial x_2} +,...,+ \dfrac{\partial V_n(x)}{\partial x_n} = \\ -\left(a_{n1} - \sum_{k=1}^{m} b_{nk}k_{k1}\right)x_1 - \left(a_{n2} - \sum_{k=1}^{m} b_{nk}k_{k2}\right)x_2 -,...,-\left(a_{nn} - \sum_{k=1}^{m} b_{nk}k_{kn}\right)x_n \end{cases} \quad (6)$$

From here we can find the components of the gradient vector for the component vector functions

$$\left(V_1(x_1, x_2,..., x_n), V_2(x_1, x_2,..., x_n),..., V_n(x_1, x_2,..., x_n)\right)$$

$$\begin{cases} \dfrac{\partial V_1(x)}{\partial x_1} = -\left(a_{11} - \sum_{k=1}^{m} b_{1k}k_{k1}\right)x_1, \dfrac{\partial V_1(x)}{\partial x_2} = -\left(a_{12} - \sum_{k=1}^{m} b_{1k}k_{k2}\right)x_2, \\ ..., \dfrac{\partial V_1(x)}{\partial x_n} = -\left(a_{1n} - \sum_{k=1}^{m} b_{1k}k_{kn}\right)x_n \\ \dfrac{\partial V_2(x)}{\partial x_1} = -\left(a_{21} - \sum_{k=1}^{m} b_{2k}k_{k1}\right)x_1, \dfrac{\partial V_2(x)}{\partial x_2} = -\left(a_{22} - \sum_{k=1}^{m} b_{2k}k_{k2}\right)x_2, \\ ..., \dfrac{\partial V_2(x)}{\partial x_n} = -\left(a_{2n} - \sum_{k=1}^{m} b_{2k}k_{kn}\right)x_n \\ ... \\ \dfrac{\partial V_n(x)}{\partial x_1} = -\left(a_{n1} - \sum_{k=1}^{m} b_{nk}k_{k1}\right)x_1, \dfrac{\partial V_n(x)}{\partial x_2} = -\left(a_{n2} - \sum_{k=1}^{m} b_{nk}k_{k2}\right)x_2, \\ ..., \dfrac{\partial V_n(x)}{\partial x_n} = -\left(a_{nn} - \sum_{k=1}^{m} b_{nk}k_{kn}\right)x_n \end{cases} \quad (7)$$

Total time derivative of the components of the vector Lyapunov function $V_i(x)$ given by the equation of motion (1) and (4) is determined by

$$\frac{dV_i(x)}{dt} = -\begin{bmatrix} \left(a_{i1} - \sum_{k=1}^{m} b_{ik}k_{k1}\right)x_1 + \left(a_{i2} - \sum_{k=1}^{m} b_{ik}k_{k2}\right)x_2 + \\ +,,,+\left(a_{in} - \sum_{k=1}^{m} b_{ik}k_{kn}\right)x_n \end{bmatrix}^2, \quad (8)$$

$i = 1,2,...,n$

From the expressions (8) that the total time derivative of the vector-Lyapunov $V_i(x)$ functions in the performance of the initial assumptions resulting from the geometric interpretation of a theorem A.M. Lyapunov will be negative sign function. This means that the conditions for asymptotic stability of the system will always be performed (4).

Now, using components of the gradient vector we will restore components of the vector Lyapunov functions:

$$V_i(x_1, x_2,..., x_n) = -\left(a_{i1} - \sum_{k=1}^{m} b_{ik}k_{k1}\right)x_1^2 - \left(a_{i2} - \sum_{k=1}^{m} b_{ik}k_{k2}\right)x_2^2 -,$$
$$-...,-\left(a_{in} - \sum_{k=1}^{m} b_{ik}k_{kn}\right)x_n^2,$$
$$i = 1,2,...,n$$

The positive definiteness of all components of the vector Lyapunov function will be expressed by

$$-\left(a_{ij} - \sum_{k=1}^{m} b_{ik}k_{kj}\right) > 0, i = 1,2,...,n, j = 1,2,..,n \quad (9)$$

This condition characterized superstability of transposed matrix of a closed system [4].

IV. RADIUS OF THE ROBUSTNESS

A. *Define the Radius of the Robustness.*

Let us investigate the robust stability of the vector-Lyapunov functions. Then let us transform the condition of robust stability of the components of the vector Lyapunov function. For this, we can turn to a parametric family of coefficients the vector-Lyapunov functions, such as the interval family, defined as [4]:

$$d_{ij} = d_{ij}^0 + \Delta_{ij}, |\Delta_{ij}| \leq \gamma\, m_{ij}, i, j = 1,2,...,n$$

where the nominal rate

$$d_{ij}^0 = -\left(a_{ij}^0 - \sum_{k=1}^{m} b_{ik}^0 k_{kj}^0\right)$$

corresponds to a positive-definite Lyapunov functions, i.e.

$$\sigma(D_0) = \min_i \min_j -\left(a_{ij}^0 - \sum_{k=1}^{m} b_{ik}^0 k_{kj}^0 \right) > 0$$

Now, we require that the positivity condition coefficients stored for all functions of the family:

$$-\left(a_{ij}^0 - \sum_{k=1}^{m} b_{ik}^0 k_{kj}^0 \right) + \Delta_{ij} > 0, i = 1,2,...,n;\ j = 1,2,...,n$$

Clearly, this inequality holds for all admissible Δ_{ij} if and only if

$$-\left(a_{ij}^0 - \sum_{k=1}^{m} b_{ik}^0 k_{kj}^0 \right) + \gamma m_{ij} > 0,$$

$$i = 1,2,...,n;\ j = 1,2,...,n$$

i.e. when

$$\gamma < \gamma^* = \min_i \min_j \frac{-\left(a_{ij}^0 - \sum_{k=1}^{m} b_{ik}^0 k_{kj}^0 \right)}{m_{ij}} \qquad (10)$$

In particular, if $m_{ij} = 1$ (scale factors of a member of Lyapunov functions are the same), then

$$\gamma^* = \sigma(D_0) \qquad (11)$$

Thus, the stability radius of interval family of positive definite functions is the smallest value of the coefficients of the vector Lyapunov functions.

B.Example for Second Order System

As an example, we consider the second order system.
Let *n = 2, m = 1*, i.e.,

$$\dot{x} = Ax + Bu,\ A = \begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix},$$

$$B = b = \begin{Vmatrix} b_1 \\ b_2 \end{Vmatrix},\ u = -Kx, \quad K = k = \begin{Vmatrix} k_1 & k_2 \end{Vmatrix}$$

$$\dot{x}_1 = (a_{11} - b_1 k_1) x_1 - (a_{12} - b_1 k_2) x_2$$
$$\dot{x}_2 = (a_{21} - b_2 k_1) x_1 - (a_{22} - b_2 k_2) x_2 .$$

Then

$$G = A + BK = \begin{Vmatrix} a_{11} - b_1 k_1 - (a_{12} - b_1 k_2) \\ a_{21} - b_2 k_1 - (a_{22} - b_2 k_2) \end{Vmatrix}$$

With inequality [4] characteristic equation has roots with negative real parts.

$$\begin{cases} a_{22} - b_2 k_2 - a_{11} + b_1 k_1 > 0 \\ (a_{12} - b_1 k_2)(a_{21} - b_2 k_1) - (a_{11} - b_1 k_1)(a_{22} - b_2 k_2) > 0 \end{cases}$$

We investigate the stability of the system using the idea of Lyapunov functions.

Let us investigate the components of the gradient vector components vector functions $V_1(x_1, x_2)$ and $V_2(x_1, x_2)$:

$$\frac{\partial V_1(x_1,x_2)}{\partial x_1} = -(a_{11} - b_1 k_1) x_1,\ \frac{\partial V_1(x_1,x_2)}{\partial x_2} = +(a_{12} - b_1 k_2) x_2$$
$$\frac{\partial V_2(x_1,x_2)}{\partial x_1} = -(a_{21} - b_2 k_1) x_1,\ \frac{\partial V_2(x_1,x_2)}{\partial x_2} = +(a_{22} - b_2 k_2) x_2$$

We discover the total time derivative of the Lyapunov function by the formula (8):

$$\frac{dV(x_1,x_2)}{dt} = -\left[(a_{11} - b_1 k_1) x_1 + (a_{12} - b_1 k_2) x_2\right]^2 - $$
$$-\left[(a_{21} - b_2 k_2) x_1 + (a_{22} - b_2 k_2) x_2\right]^2 < 0$$

The next step - discovering vector Lyapunov functions

$$V_1(x_1,x_2) = -\frac{1}{2}(a_{11} - b_1 k_1) x_1^2 + \frac{1}{2}(a_{12} - b_1 k_2)_2^2$$
$$V_2(x_1,x_2) = -\frac{1}{2}(a_{21} - b_2 k_1) x_1^2 + \frac{1}{2}(a_{22} - b_2 k_2) x_2^2$$

Conditions for the stability of the system obtained in the form:

$$-(a_{11} - b_1 k_1) > 0,\ (a_{12} - b_1 k_2) > 0,$$
$$-(a_{21} - b_2 k_1) > 0,\ (a_{22} - b_2 k_2) > 0$$

and

$$-(a_{11} - b_1 k_1) > (a_{21} - b_2 k_1),$$
$$(a_{22} - b_2 k_2) > -(a_{12} - b_1 k_2)$$

From this we can get a system of inequalities

$$a_{22} - b_2 k_2 - a_{11} + b_1 k_1 > 0$$
$$(a_{12} - b_1 k_2)(a_{21} - b_2 k_1) - (a_{11} - b_1 k_1)(a_{22} - b_2 k_2) > 0$$

Thus, from (9) and (10) we can determine the radius of robust stability of a second order system, if system parameters are uncertain:

$$\gamma^* = \min \begin{Bmatrix} -(a_{11} - b_1 k_1), (a_{12} - b_1 k_2), \\ -(a_{21} - b_2 k_1), (a_{22} - b_2 k_2) \end{Bmatrix}$$

C.Experimental results

Then, as an example, we define the following initial conditions, find conditions for the stability of the second order system and the radius and transients.

When the initial settings are follow:

$$A = \begin{Vmatrix} -11.6 & 45 \\ 11.7 & 0.4 \end{Vmatrix}, B = \begin{Vmatrix} 9.2 \\ 7 \end{Vmatrix}, K = \begin{Vmatrix} 2 & 0.001 \end{Vmatrix}$$

In this case, the radius will be equal ($\gamma^* = 0.3860$).

The overall the transition process of the system shows on the Figure 2.

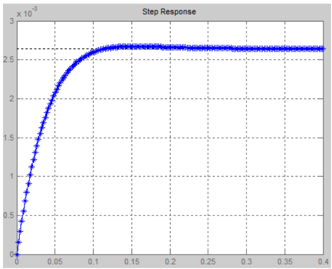

Fig. 2.The transition process, exp.1.

The second case, when the initial settings are follow:

$$A = \begin{Vmatrix} -11.6 & 45 \\ 11.7 & 0.4 \end{Vmatrix}, B = \begin{Vmatrix} 9.2 \\ 7.3 \end{Vmatrix}, K = \begin{Vmatrix} 2 & 0.001 \end{Vmatrix}$$

In this case, the radius will be equal ($\gamma^* = 0.3854$).

The overall the transition process of the system shows on the Figure 3.

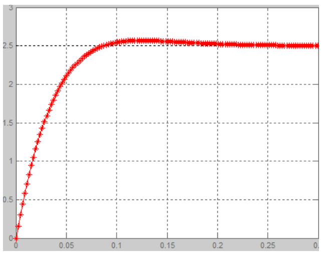

Fig. 3.The transition process, exp.2.

The third case, when the initial settings are follow:

$$A = \begin{Vmatrix} -11.6 & 55 \\ 11.7 & 0.4 \end{Vmatrix}, B = \begin{Vmatrix} 9.2 \\ 7.3 \end{Vmatrix}, K = \begin{Vmatrix} 2 & 0.003 \end{Vmatrix}$$

In this case, the radius will be equal ($\gamma^* = 0.3562$).

The overall the transition process of the system shows on the Figure 4.

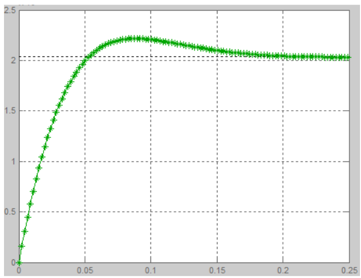

Fig. 4.The transition process, exp.3.

For 4-d case, when the initial settings are follow:

$$A = \begin{Vmatrix} -11.6 & 55 \\ 11.7 & 0.4 \end{Vmatrix}, B = \begin{Vmatrix} 9.9 \\ 7.3 \end{Vmatrix}, K = \begin{Vmatrix} 2 & 0.003 \end{Vmatrix}$$

In this case, the radius will be equal ($\gamma^* = 0.3562$).

The overall the transition process of the system shows on the Figure 5.

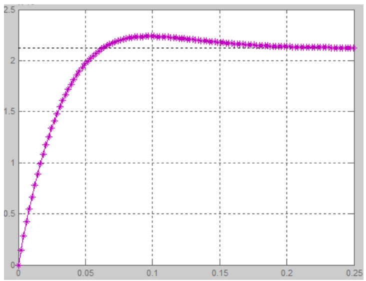

Fig. 5.The transition process, exp.4.

The short analysis of examples #3 and #4 we can see on Figure 6 and complex analysis of examples #1- #4 shown on Figure 7. For all the examples given initial values and the stability conditions of the system are executed.

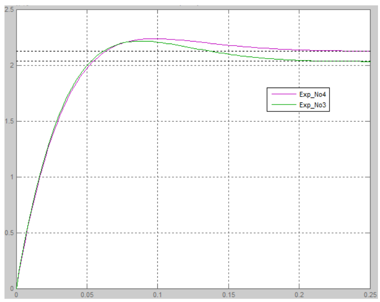

Fig. 6.The transition process, exp.3-4.

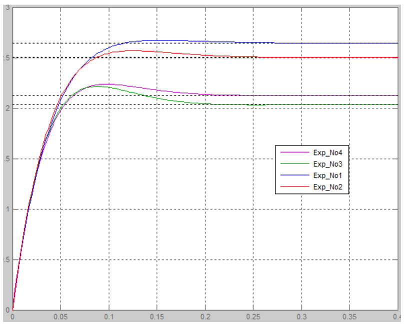

Fig. 7.The transition process, exp.1-4.

The experimental results of the proposed robust control system and comparing the satisfied results obtained.

V.CONCLUSIONS

To conclude, theory robust stability performs an important role in the theory of control of dynamic objects [6]. The main task in studying robust stability is to specify constraints on the change of control system parameters that preserve stability. These restrictions determined by the range of stability at undetermined and selected, i.e. varying parameters [7,8].

This work provides for construction of Lyapunov function in a form of vector-function so that its gradient shall be equal to components of velocity vector (right side of the equation) but with opposite sign. The study of robust stability of the system is founded on the basis of Lyapunov's direct method. Stability region resulted in simplest equations with undetermined parameters of control objects and selected parameters of regulator.

This article described geometric interpretation of Lyapunov function in detail. Conditions of robust stability were identified. Robust stability of Lyapunov function was examined by applying interval matrix family of coefficients. The radius of stability coefficients interval family of positive definite functions is equal to the smallest value of the coefficients of the vector Lyapunov functions. Detailed case study was presented. Besides, theoretical results obtained in this paper provide considerable contribution to the theory of stability at certain indefiniteness, i.e. to the theory of robust stability of linear control systems .

Practical importance of achieved results from one side provides for application of common applied approaches to define region of robust stability [9-12] that will preserve dynamic safety of the object under investigation, from the other side these studies important for providing an opportunity to design more effective control systems [13-15].

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