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Currency Portfolio Investment Hedged with Currency Basket Options

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Abstract— **We provide a nonparametric ordering of competing strategies to hedge multidimensional foreign exchange rate risk based on stochastic dominance tests. The hedging strategies include basket options, standard forward and option contracts and a selective hedging strategy. Empirical analysis spanning more than 14 years daily data show that basket option hedging dominates all the other hedging strategies. The finding holds for an equally weighted and two mean variance optimized portfolios for investment horizons of three, six and twelve months.**

Keywords— **Basket options, Quasi-Monte Carlosimulation, Stochastic dominance, Hedging.**

I. Introduction

The increased globalization of the financial market has allowed multinational companies and investors to extend their exposures to a wide range of foreign assets with different invoice currencies. International financial integration has also boosted financial innovation which is best reflected in a set of new risk management vehicles. Consequently, by holding a well diversified portfolio of financial assets, global investors endure the main issue of choosing the suitable risk management tool.

It is well recognized that hedging can affect portfolio performance and firm value [20]. Moreover, the selection of the appropriate risk management tool is a crucial task in the risk management process and significantly impact the firm value [13].The complexity of the choice among the hedging tools arises in defining the nature of the exposure, in accurately measuring the risk exposure and in deciding on the appropriate degree of risk exposure that ought to be hedged. A hedger should choose a hedge instrument that matches the risk profile of the underlying currency position as closely as possible.

In this paper we apply stochastic dominance tests to compare the performance of different strategies to hedge foreign exchange rate risk. If the underlying currencies to hedge are correlated, a significant cost reduction can be achieved through the use of basket options instead of hedging each currency apart. Given that the joint probability of the sum of lognormally distributed variables does not have an explicit formula, a numerical solution using Quasi-Monte Carlo method is proposed to approximate the fair value of the currency basket option.

We consider an investor with the United States dollar (USD) asthe reference currency who isexposed to the global currency risk through investment in the most active currencies in the foreign exchange market: The euro (EUR), the Japanese yen (YEN), the Swiss franc(CHF), the Great Brittan pound (GBP), the Canadian dollar (CAD) and the Australian dollar (AUD). The investigation period spans a wide period of subsequent financial crisis.

It is widely reported in financial literature that financial markets exhibit high correlation in crisis periods [21]. Thus, by the deliberate choice of both the currency as the underlying asset class and the investigation period, we aim to assess whether the attractiveness of basket options remains unaltered in the worst case of high currency market integration.

Empirical results show that integrating basket options into an unhedged portfolio significantly shift its return distribution. Moreover, a basket option hedging stochastically dominates conventional forward and option strategies and selective hedging. These results apply for the $\frac{1}{N}$ portfolio rule and the minimum variance and tangency portfolios for various investment horizons.

The layout of the paper is as follows: in the second section we derive the mathematical formulas for the hedging strategies. In the third section, we expose the basics of Quasi Monte Carlo sampling for basket option pricing. Section four exhibits stochastic dominance test for bitwise comparison of the hedging strategies. Section fivedescribes the dataset. Section sixdisplays the main findings and section seven concludes.

II. HEDGING STRATEGIES

We examine the effectiveness of the following hedging strategies:

- Unhedged strategy (UnhRet),
- Forward hedging (FRet),
- Protective put hedging (OptRet),
- Basket options hedging (Bret),
- Random walk selective hedging (RWRet).

A. *Unhedged Portfolio Returns*

The return on foreign exchange is defined as the return in investing on a foreign riskless bond. Holding an unhedged foreign currency$i$ yields the following dollar returns:

$$r_i^{\$} = (1 + e_i)(1 + r_i^f) - 1 \quad (1)$$

This can be written:

$$r_i^{\$} = r_i^f + e_i + r_i^f e_i \quad (2)$$

Wherer_i^f is the return on the foreign riskless bond.$e_i = \frac{\Delta S_t}{S_t} = \frac{S(T)-S(0)}{S(0)}$is the relative change of foreign currency i between dates *0* and *T*.$S(0)$and $S(T)$ are the observed spot exchange rates at dates 0 and T, respectively.

Aggregating equation (1) to the portfolio level gives the following dollar unhedged portfolio return:

$$R^{\$} = \sum_{i=1}^{d} w_i \left[(1 + e_i)(1 + r_i^f) - 1\right] \quad (3)$$

$w_i$is the weight of currency i in the portfolio.

This equation serves as the basis for calculating hedged returns.

B. *Forward hedging returns*

The object of forward hedge is to lock in the forward value of the currency portfolio at time 0 so that there’s no uncertainty about its value at the investment maturity T.

Despite the huge financial innovations, forwards is the widely used hedging vehicle(Jesswein et al. 1995). Forward hedging consists in holding a forward position equal to the opposite of the spot position. The return of the forward hedging strategy is described by the following equation :

$$FRet = \sum_{i=1}^{d} w_i \left[r_i^{\$} + h(f_i - e_i)\right] \quad (4)$$

Where$h$ is the hedge ratio and$f$is the forward premium (or discount). For a one to one hedge, equation (4) reduces to:

$$FRet = \sum_{i=1}^{d} w_i \left(r_i^{\$} + f_i - e_i\right) \quad (5)$$

C. *Protective put hedging*

A long put option position on a single currency i yields the following dollar hedged return:

$$OptRet_i = r_i^{\$} + \frac{Max(0, K_i - S_i(T)) - P_i}{S_i(0)} \quad (6)$$

K and P are, respectively, the option's strike price andpremium.

For a portfolio containing d currencies, hedging separately each currency position using $d$protective puts gives the following dollar portfolioreturn :

$$OptRet = \sum_{i=1}^{d} w_i \left(r_i^{\$} + \frac{Max(0, K_i - S_i(T)) - P_i}{S_i(0)}\right) \quad (7)$$

D. *Selective Hedging*

It is well establishedthat a naïve random walk rule beats many fundamental based models of exchange rate determination such as the monetary model, interest rate parity and purchasing power parity[12, 17,8, 1].Accordingly, selective hedging based on the random walk behaviour of the exchange rate can outperform passive hedging strategies [9,10].

Compared to static hedging, the random walk model is an active forward looking hedging strategy. For a single currency, the random walk rule states that the hedger short forward the currency whenever it is at a forward premium ($f > 0$). In the opposite case ($f < 0$) the currency position is kept unhedged.

E. *Basket option hedging*

Portfolios containing two or more exchange rates can be hedged using a special type of exotic derivatives called a basket option. The major advantage of basket options is that they tend to be more cost-effective than the corresponding portfolio of standard options for at least two reasons [11]: Firstly, the volatility of the basket is in most cases less than the sum of volatilities due to the fact that the underlying currencies in the basket are not perfectly correlated. Secondly, a basket option minimizes transaction and administrative costs because an investor has to buy only one option instead of several ones.

In basket options the final payoff that an option holder stands to receive depends on the performance of whole portfolio of currencies. Theexpostdollar hedged return of a put basket option strategy is given by the following equation:

$$BRet = \sum_{i=1}^{d}\left(w_i r_i^{\$} + \frac{Max(0, B_0 - w_i(0)S_i(T)) - P_B}{w_i S_i(0)}\right) \quad (8)$$

$B_0$and$P_B$are, respectively, the basket srike and premuim. We consider hedging by an at the money currency basket options for which the strike rice is given by: $B_0 = \sum_{i=1}^{d} w_i(0)S_i(0)$.

III. QUASI-MONTECARLO BASKET OPTION PRICING

Estimating the value of a currency basket option consists mainly on solving a multidimensional stochastic integral of the form:

$$P_B = e^{-r_d T}\int_{\infty} \dots \int_{\infty}\left(B_0 - \sum_{i=1}^{d} w_i(0)S(T)\right)_{+} p(\phi_1, \phi_2, \dots, \phi_d)\, d\phi_1 d\phi_2 \dots d\phi_d \quad (9)$$

$r_d$is the domestic interest rate,$p(\phi_1, \phi_2, \dots, \phi_d)d\phi_1 d\phi_2 \dots d\phi_d$is the joint probability density function of the correlated exchange rates.$\theta_i$are standard Gaussian random variates with correlations given by: $dW_i dW_j = \rho_{i,j}dt$.The dimension $d$stands for number of currencies included in basket.

We assure a Black and Sholes (1973) [3] framework for which the exchange rate process is lognormal:

$$S_i(t+\Delta t) = S_i(t)\exp\left(\left(r_d - r_i - \frac{1}{2}\sigma_i^2\right)\Delta t + \sigma\phi_i\sqrt{\Delta t}\right) \quad (10)$$

To the extent that the sum of $d$lognormally distributed variables is not lognormal, we cannot derive a closed form solution for the price of the basket option. Standard Monte Carlo Methods are currently used to compute a numerical approximation of the basket option price. However, these methods suffer from a low convergence rate of about $\sigma(d^{-\frac{1}{2}})$. That is, 100 supplementary replications are needed to increase the estimator accuracy by a factor of 10. The low convergence rate is attributed to the low uniformity of random points in the unit hypercube. To circumvent this pricing problem we use Quasi Monte Carlo numerical integration. Quasi Monte Carlo is a powerful algorithm for dealing with the pricing of multidimensional contingent claims. Instead of generating random variates for numerical integration, Quasi-Monte Carlo proceeds with the use of deterministic sequences or sequences of low discrepancy. The discrepancy of points is a measure of the uniformity of points in the unit hypercube domain. The more the points are uniform, the less is the discrepancy. Increased uniformity results on improved convergence rate of the Quasi Monte Carlo method over the standard Monte Carlo Method.It has been show in [18] and [6]that Quasi Monte Carlo convergence rate is about $\sigma(d^{-1})$ and can reach$\sigma(d^{-\frac{3}{2}})$ in some cases. The most familiar sequences used in the pricing of multidimensional contingent claims are the Sobol, Halton and Faure sequences [4].

Figure 1 displays the distribution of points in the unit cube domain using a random numbers generator and the Sobol low discrepancy sequence. Points used by the Sobol sequence are more uniform than points pick from a random number generator. New added points, for the case of the low discrepancy points, progressively fill the gap between previous points. In the case of the random number generator, points newly generated use to cluster.

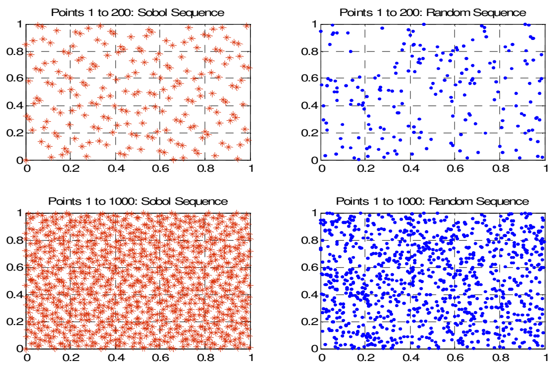

Figure 1: Distribution of the Sobol vs random points in the $[0,1] \times [0,1]$rectangle

Based on empirical findings that the Sobol method outperforms the basic Monte Carlo and many deterministic sequences for the pricing of multidimensional contingent claims [15,19] we use this sequence to price a currency call basket option.

IV. NON PARAMETRIC ORDERING OF THE HEDGING STRATEGIES

Due to the presence of option related strategies that can alter the distribution of the portfolio returns [5] we propose to use stochastic dominance as a distribution free performance appraisal approach.

Let$\{Y_i\}_{i=1}^{N}$and$\{Z_i\}_{i=1}^{M}$ two independent random samples from two hedging strategies F and G with common support r and empirical cumulative distributions given by:

$$\hat{F}_N = \frac{1}{N}\sum_{i=1}^{N} I(Y_i \le r)\ ,\ \hat{G}_N = \frac{1}{M}\sum_{i=1}^{M} I(Z_i \le r),$$

with$I(.)$ denote the indicator function.

Let $D_s(:, F)$ the function that integrates $F$to order s-1:

$$D_1(:,F) = F(r)$$

$$D_2(:,F) = \int_0^x F(t)d(t) = \int_0^x D_1(t,F)dt$$

$$D_3(:,F) = \int_0^x \int_0^t F(s)dsd(t)$$

$D_s(:,G)$is defined analogously.
The null (and alternative) of stochastic dominance of order s is written as:

$$\begin{cases} H_0^s: D_s(r_k,F) \leq D_s(r_k,G) \text{against} \\ H_1^s: D_s(r_k,F) > D_s(r_k,G) \end{cases}$$

Barrett and Donald [2] propose the following test statistic that can be estimated using the boostrap method.

$$\hat{S}_s = \left(\frac{NM}{N+M}\right)^{\frac{1}{2}} sup_x \left(D_s(r_k,\hat{F}_N) - D_s(r_k,\hat{G}_M)\right)(11)$$

V. DATA DESCRIPTION

Data are drawn from the federal reserve bank of Saint Louis. These include daily exchange rates against the USD (EUR, JPY, CHF, GBP and CAD) and daily Libors on the corresponding currencies.The data range from the first of January 2001 to 30 May 2014. We assume aone quarter investment horizon in the equally weighted portfolio, similar results are obtained for 6 and 12 months horizons in the minimum variance and tangency portfolios[1].

VI. EMPIRICAL RESULTS

Table 1displays some descriptive statistics on the return distribution of the competing hedging strategies.

TABLE I
SUMMARY STATISTICS FOR DAILY RETURNS ON THE HEDGING STRATEGIES

	FRet	OptRet	UnhRet	RWRet	BRet
Mean (%)	2.381	3.4125	3.409	3.558	4.151
Max (%)	5.134	25.766	16.730	14.781	14.692
Min (%)	0.622	-12.88	-12.831	-13.493	-1.404
Volatility (%)	1.215	6.417	4.218	3.478	2.954
Sharpe	1.959	0.531	0.808	1.023	1.405
Skewness	0.301	0.390	-0.153	-0.772	0.599
Kurtosis	1.952	2.649	2.982	4.383	2.700
Jarque-Bera	180.43 (0)	90.57 (0)	11.618 (0.003)	530.73 (0)	188.39 (0)

Note on table: numbers in parenthesis are p-values

[1]Results are available from the authors upon request.

The basket option hedging strategy exhibit the high daily return. Compared to the unhedged portfolio, integrating basket options improves the mean portfolio return and lower its daily variance. On an annual basis, the excess return of basket options hedging over the unhedged strategy is about 267%.

For investors having preference towards positive skewness, basket option is the appropriate strategy. The Sharpe ration performance criteria show that the forward hedging is the best strategy followed by basket options strategy.

Table 2 gives p-values for the differences between the Sharpe ratios computed using block bootstrap method [16]. This estimation method accounts for the existence of serial correlation in the return distributions of the hedging strategies.

TABLE II
BLOCK BOOTSTRAP SHARPE RATIO DIFFERENCE TESTS

	FRet	OptRet	UnhRet	RWRet	BRet
FRet					
OptRet	1.427 (0.056)				
UnhRet	1.151 (0.112)	-0.276 (0.002)			
RWRet	0.936 (0.105)	-0.491 (0.154)	-0.214 (0.626)		
BRet	0.554 (0.1148)	-0.873 (0.101)	-0.596 (0.138)	-0.382 (0.345)	

Note on table: The table contain the difference of Sharpe ratio of strategy in column minus Sharpe ratio of strategy in line and reports p-values (in parenthesis) for testing this differences using block bootstrap estimation.

Except for the case the unhedging versus standard option hedging all differences in the Sharpe ratios are statistically different from zeros.

By restricting the performance comparison to the two first moments of the return distribution, forward hedging exhibit significant high Sharpe ratio over all the hedging strategies. However, as shown in table 1 the Gaussian assumption is significantly rejected by the data. This results on biased ranking of the hedging strategies

Table 3 reportsBarret and Donald [2] bootstrapped p-values.

When extending performance comparison to the whole distribution of the hedging returns, the ranking of the competing strategies differ significantly from linear mean variance performance measure. As table 3 shows, the basket option stochastically dominates forward hedging at the third order. This finding is in contradiction with the Sharp ratio comparison. Investors exhibiting preference towards positive skewness will find basket options more adequate than forward hedging.

Integrating the basket options into an unhedged portfolio shifts significantly the return distribution of the portfolio return. A portfolio containing basket options dominates at second order an unhedged portfolio with p-value equal to 0.8. Thus, risk adverse investors will prefer to hedge their currency exposure with basket options instead of keeping unhedged their portfolios.

TABLE III
STOCHASTIC DOMINANCE TEST

	SDj	FRet	OptRet	UnhRet	RWRet	BRet
FRet	**SD1**		0	0	0	0
	SD2		0	0	0	0
	SD3		0	0	0	0.4
OptRet	**SD1**	0		0	0	0
	SD2	0		0	0	0.6
	SD3	0		0.8	0.2	1
UnhRet	**SD1**	0	0		0	0
	SD2	0	0		0	0.8
	SD3	0	0		0	0.6
RWRet	**SD1**	0	0	0		1
	SD2	0	0	0		1
	SD3	0	0	0.2		1
BRet	**SD1**	0	0	0	0	
	SD2	0	0	0	0	
	SD3	0	0	0	0	

VII. CONCLUSIONS

In this paper we propose a range of linear and non-linear currency hedging strategiesfor international currency portfolio investment. Instead of hedging each currency separately it is more appropriate to adopt a portfolio approach that takes into account the correlations between the different currencies. To attain this goal, basket options are an efficient tool. Given the non-linear payoff structure of options we applied stochastic dominance to rank the different hedging strategies. Empirical results show that basket option hedging dominate all the hedging strategies for different stochastic dominance orders. The finding holds for both the equally weighted portfolio and two mean variance optimized portfolios.

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