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Existence of the solution of a quasilinear equation and its application to image denoising

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***Abstract*—We give in this paper a new method to show the existence of the solution for the proposed model is a combination of the Perona-Malik equation and the Heat equation. We also give numerical implementation details and show experimental results on examples images which prove the efficiency and effectiveness of our model.**

***Index Terms*—Perona-Malik equation, functional minimization, quasi-linear equation, image denoising, image decomposition**

I. Introduction

The purpose of this article is to investigate the existence of solution for quasilinear equation, in a bounded domain of $\mathbb{R}^N$, with Neumann boundary conditions. The stady of this problem started with the use of Brouwer degree theory, then we pass to the limit when we do reach dimension approximation spaces to infinity to build a solution the starting problem. We also study the numerical of our problem and we prove that the limit problem coincides with the Perona-Malik model (see [2], [11]) in the some subregion where the proposed model is an interpolation of two classical models, Perona-Malik [1], [3], [7] and the Heat equation, as in [13] where they proposed a modified Perona-Malik model based on directional Laplacian, for alleviate the staircasing effect, preserve sharp discontinuities, and remove noise simultaneously. On the other hand there are several new works on the same field used the method of the total variation (see [8], [9], [10]). In this work we establish the existence of weak solution of the problem

$$\begin{cases} -\operatorname{div}\Big(g(|\nabla u|)\nabla u\Big) - \dfrac{1}{\lambda^2}\operatorname{div}(\nabla u) = f(x) - uk(x) \ \text{ in } \Omega, \\ \Big(g(|\nabla u|) + \dfrac{1}{\lambda^2}\Big)\nabla u \cdot \vec{\eta} = 0. \ \text{ on } \partial\Omega, \end{cases} \tag{1}$$

where f is a given function, $\Omega \subseteq \mathbb{R}^N$ is the bounded domain with smooth boundary $\partial\Omega$, $\lambda > 1$ be a given contrast parameter, $k \in L^\infty(\Omega)$ and the function $g(\cdot)$ is defined by the following expression:

$$g(z) = \frac{1}{1+(\frac{z}{\lambda})^2} \quad \text{or} \quad g(z) = \exp\big(-\frac{z^2}{2\lambda^2}\big).$$

It is clear, that the function $g(z)$ is a decreasing non-negative function satisfies the following conditions

$$\begin{cases} \lim\limits_{z\to 0} g(z) = 1, \\ \lim\limits_{z\to +\infty} g(z) = 0. \end{cases} \tag{2}$$

In the case where the Euler-Lagrange equation equal to $u - \operatorname{div}\Big(g(|\nabla u|)\nabla u\Big) - \dfrac{1}{\lambda^p}\operatorname{div}(|\nabla u|^{p-2}\nabla u) = f(x)$ was treated in [2].
The problem (1) is equivalent to solve the Perona-Malik problem in the region where the norm of the gradient is less than λ and equivalent to solve the heat equation if not.

This paper is organized as follows. In the next section, we prove the existence of the solution of the problem (1). And the last section is devoted to numerical results and comments.

II. Proposed model

Given $f \in L^2(\Omega)$ and $k \in L^\infty(\Omega)$, in the application k is the probability density of noise (we speak here on the gaussian noise because the probability density of this variable is the gaussian law). We are interested in finding weak solutions of the problem (1) for a quasilinear equation, we need the following definition

Definition II.1. *We say $u \in H^1(\Omega)$ is a weak solution for the problem* (1) *if for any $\varphi \in H^1(\Omega)$ we have*

$$\int_\Omega \Big(g(|\nabla u|)+\frac{1}{\lambda^2}\Big)\nabla u \nabla\varphi \,\mathrm{d}x = \int_\Omega f(x)\varphi\,\mathrm{d}x - \int_\Omega uk(x)\varphi\,\mathrm{d}x. \tag{3}$$

We use the same notations as in the previous section.

Theorem II.1. *Under condition* (2)*, problem* (1) *has at least one solution.*

Proof: Let V be a finite-dimensional subspace of $H^1(\Omega)$ endowed with the H^1-norm, and V^* its dual. Define the

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mappings $H : V \times [0,1] \longrightarrow V^*$ by

$$\begin{aligned} &\langle H(u,t),\varphi\rangle_H \\ &= \int_\Omega \Big(g(t|\nabla u|) + \frac{1}{\lambda^2}\Big)\nabla u \nabla\varphi \,\mathrm{d}x - \int_\Omega f(x)\varphi\,\mathrm{d}x \\ &+ \int_\Omega uk(x)\varphi\,\mathrm{d}x, \end{aligned} \tag{4}$$

for all $\varphi \in V$, H is well defined. Let us show now that

$$\Big\{u \in V : H(u,t) = 0, \quad \text{for some } t \in [0,1]\Big\} \subset \bar{B}(0,\widetilde{\rho}).$$

such that

$$\widetilde{\rho} = \frac{1}{\min(\frac{1}{\lambda^2},k)}\|f\|_{L^2}$$

Lemma II.1. *Let the mappings H defined by* (4)*, there exist $R > 0$, such that*

1)
$$\begin{cases} \forall t \in [0,1], \forall u \in V \\ H(t,u) = 0 \Rightarrow \|u\|_{H^1(\Omega)} \le R. \end{cases}$$
2) *H is bounded.*
3) *H is continuous on $\bar{B}^{\mathrm{V}}(R) \times [0,1]$.*

Proof:
Indeed, if $H(u,t) = 0$ for some $(u,t) \in V \times [0,1]$, then

$$\begin{aligned} \|f\|_{L^2}\|u\|_{H^1(\Omega)} &\ge \frac{1}{\lambda^2}\int_\Omega |\nabla u|^2\,\mathrm{d}x + \int_\Omega u^2 k(x)\,\mathrm{d}x \\ &\ge \min\Big(\frac{1}{\lambda^2},k\Big)\|u\|^2_{H^1(\Omega)}, \end{aligned}$$

which implies that

$$\|u\|_{H^1(\Omega)} \le \frac{1}{\min\Big(\frac{1}{\lambda^2},k\Big)}\|f\|_{L^2}. \tag{5}$$

Consequently, for any $R > \Big(\min(\frac{1}{\lambda^2},k)\Big)^{-1}\|f\|_{L^2}$ we have

$$H(u,t) \neq 0 \quad \text{if } (u,t) \in \partial B^V(R) \times [0,1].$$

We now show that the mapping H is bounded, if $(u,t) \in \bar{B}^V \times [0,1]$, we have

$$\begin{aligned} &|\langle H(u,t),\varphi\rangle| \\ &\le \Big(\underbrace{\max\Big(1+\frac{1}{\lambda^2},\|k\|_{L^\infty}\Big)R + \|f\|_{L^2}}_{\widetilde{R}}\Big)\|\varphi\|_{H^1(\Omega)}, \end{aligned}$$

for all $\varphi \in H^1(\Omega)$, and hence

$$H\Big(\bar{B}^{\mathrm{V}}(R)\times[0,1]\Big) \subset \bar{B}^{\mathrm{V}^*}(\widetilde{R}). \tag{6}$$

We now show that H is continuous on $\bar{B}^{\mathrm{V}}(R) \times [0,1]$.
Let $(u_n,t_n) \in \bar{B}^{\mathrm{V}}(R) \times [0,1]$ converge to (u,t) in $V \times [0,1]$, i.e in $H^1 \times [0,1]$. Since $(H(u_n,t_n))$ is bounded because of (6), to prove that

$$H(u_n,t_n) \to H(u,t)$$

it is sufficient to show that $H(u,t)$ is the unique cluster point of $(H(u_n,t_n))$. Let $\widetilde{k} \in \mathrm{V}^*$ be such a cluster point, still denoted by $(t_n),(u_n)$ a subsequence of $(t_n),(u_n)$ respectively such that

$$H(u_n,t_n) \to \widetilde{k} \text{ in } \mathrm{V}^*.$$

Since $u_n \to u$ in $H^1(\Omega)$, it follows that $u_n \to u$ in $L^2(\Omega)$, and hence, going if necessary to a subsequence, we may assume that $u_n \to u$ a.e in Ω . On the other hand, $\partial_i u_n \to \partial_i u$ in $L^2(\Omega)$, therefor $\nabla u_n \to \nabla u$ a.e in Ω. This implies that

$$g(t_n|\nabla u_n|) \to g(t|\nabla u|) \quad \text{a.e in } \Omega, \tag{7}$$

and hence, for any $\varphi \in V$,

$$g(t_n|\nabla u_n|)\nabla\varphi \to g(t|\nabla u|)\nabla\varphi$$

in $L^2(\Omega)$. We conclude that

$$\begin{aligned} &\langle H(t_n,u_n),\varphi\rangle_H \\ &= \int_\Omega u_n k(x)\varphi\,\mathrm{d}x + \int_\Omega \Big(g(t_n|\nabla u_n|) + \frac{1}{\lambda^2}\Big)\nabla u_n\nabla\varphi\,\mathrm{d}x \\ &\to \int_\Omega uk(x)\varphi\,\mathrm{d}x + \int_\Omega \Big(g(t|\nabla u|) + \frac{1}{\lambda^2}\Big)\nabla u\nabla\varphi\,\mathrm{d}x \\ &= \langle H(t,u),\varphi\rangle_H. \end{aligned}$$

Thus $\widetilde{k} = H(t,u)$. ■

It is clear that

$$H : V \times [0,1] \to V^*$$

is a continuous homotopy and the existence of at least one solution of the problem (1) would follow from

$$\deg_B\Big(H(\cdot,1),B(R),0\Big) \neq 0$$

All those proprieties allow us to apply the homotopy invariance propriety and obtain

$$\deg_B\Big(H(\cdot,1),B(R),0\Big) = \deg_B\Big(H(\cdot,0),B(R),0\Big). \tag{8}$$

But $H(u,0) = 0$ is equivalant to the linear problem

$$(1+\frac{1}{\lambda^2})\int_\Omega \nabla u\nabla\varphi\,\mathrm{d}x - \int_\Omega f(x)\varphi\,\mathrm{d}x + \int_\Omega uk(x)\varphi\,\mathrm{d}x = 0,$$

for all $\varphi \in \mathrm{V}$, whose solution is unique because of the bounededness of the set of its possible solutions. Consequently,

$$\deg_B\Big(H(\cdot,0),B(R),0\Big) = \pm 1$$

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and from (8) and the existence propriety of degree, there exists $u \in B^{\mathrm{V}}(R)$ wich satisfies

$$\begin{aligned}&\int_\Omega \Big(g(|\nabla u|)+\frac{1}{\lambda^2}\Big)\nabla u \nabla\varphi \,\mathrm{d}x\\ &=\int_\Omega f(x)\varphi\,\mathrm{d}x-\int_\Omega uk(x)\varphi\,\mathrm{d}x \text{ and}\\ &\|u\|_{H^1}\le \frac{1}{\min\Big(\frac{1}{\lambda^2},k\Big)}\|f\|_{L^2}\end{aligned} \tag{9}$$

for all $\varphi \in V$. We now show the passage to the limit. Consider the function $a:\mathbb{R}^N\to\mathbb{R}^N$ defined by

$$a(\xi)=\Big(g(\xi)+\frac{1}{\lambda^2}\Big)\xi \quad \text{for any } \xi\in\mathbb{R}^N.$$

To prove the passage to the limit, we need the following two lemmas:

Lemma II.2. *Let $0<\lambda\le 1$, for any $\xi,\eta\in\mathbb{R}^N$ sush that $\xi\neq\eta$ we have*

$$\big(a(\xi)-a(\eta)\big)\cdot(\xi-\eta)>0.$$

Proof: The proof of this lemma remains to prove that F_λ is a nondecreasing function defined by

$$F_\lambda(s)=sg(s)+\frac{s}{\lambda^2}\qquad \text{for } s>0.$$

We compute for this the derivative of $F_\lambda(s)$, and then we find

$$F'_\lambda(s)=\widetilde{g}(s)+\frac{1}{\lambda^2},$$

where $\widetilde{g}(s)=\dfrac{\lambda^2-s^2}{\lambda^2(1+(\frac{s}{\lambda})^2)^2}$
or $\widetilde{g}(s)=\dfrac{\lambda^2-s^2}{\lambda^2}\exp\Big(-\dfrac{s^2}{2\lambda^2}\Big)$.
For $s\le\lambda$, we have $1-\Big(\dfrac{s}{\lambda}\Big)^2\ge 0$ and $\dfrac{\lambda^2-s^2}{\lambda^2}\ge 0$ then $F'_\lambda(s)\ge 0$.
For $s\ge\lambda$, using the fact that $\lambda^2\le 1$, we have $\dfrac{s^4}{\lambda^6}\ge\dfrac{s^2}{\lambda^2}$, we deduce that $F'_\lambda(s)\ge 0$ and we find the desired result. ■

Lemma II.3.

$$\begin{cases} \textit{If } a\in C(\mathbb{R}^N,\mathbb{R}^N),\ a(\xi)\le(1+\frac{1}{\lambda^2})\xi \quad \textit{for all } \xi\in\mathbb{R}^N\\ \textit{and}\\ \textit{if } u_n\to u \textit{ in } H^1(\Omega)\\ \textit{so}\\ a(\nabla u_n)\to a(\nabla u) \textit{ in } L^2(\Omega).\end{cases}$$

Lemma (II.3) is proved by the dominated convergence theorem of Lebesgue. Now, it is well known that one can write $H^1(\Omega)=\overline{\bigcup_{n\ge1}V_n}$ where $V_n\subset V_{n+1}(n\ge 1)$ and V_n has dimension n. Consequently, given any $\varphi\in H^1(\Omega)$, there exists a sequence φ_n with $\varphi_n\in V_n$ which converges to φ. On the other hand, by (9) applied to $V=V_n$, there exists, for each $n\ge1$, some $u_n\in V_n$ such that

$$\int_\Omega a(\nabla u_n)\nabla\psi\,\mathrm{d}x=\int_\Omega f(x)\psi\,\mathrm{d}x-\int_\Omega u_n k(x)\psi\,\mathrm{d}x,$$

for all $\psi\in V_n$. In particular, taking $\psi=\varphi_n$ introduced above,

$$\begin{aligned}&\int_\Omega a(\nabla u_n)\nabla\varphi_n\,\mathrm{d}x=\int_\Omega f(x)\varphi_n\,\mathrm{d}x-\int_\Omega u_n k(x)\varphi_n\,\mathrm{d}x,\\ &\|u_n\|_{H^1(\Omega)}\le\frac{1}{\min\Big(\frac{1}{\lambda^2},k\Big)}\|f\|_{L^2}\end{aligned} \tag{10}$$

for all $n\ge1$. The estimate in (10) implies that, going if necessary to subsequences, we can assume that there exists $u\in H^1(\Omega)$ such that

$$\begin{aligned}&u_n\to u \text{ weakly in } H^1(\Omega),\\ &u_n\to u \text{ strongly in } L^2(\Omega).\end{aligned}$$

As $\big(a(\nabla u_n)\big)_{n\in\mathbb{N}}$ is bounded in $L^2(\Omega)$, then there exists $\zeta\in L^2(\Omega)$ such that

$$a(\nabla u_n)\to\zeta \text{ weakly in } L^2(\Omega),$$

and $\nabla\varphi_n\to\nabla\varphi$ strongly in $L^2(\Omega)$, one can let $n\to\infty$ in (10) to obtain

$$\int_\Omega\zeta\nabla\varphi\,\mathrm{d}x=\int_\Omega f(x)\varphi\,\mathrm{d}x-\int_\Omega uk(x)\varphi\,\mathrm{d}x\quad \text{for all }\varphi\in H^1(\Omega). \tag{11}$$

It remains to show that

$$\int_\Omega\zeta\nabla\varphi\,\mathrm{d}x=\int_\Omega a(\nabla u)\nabla\varphi\,\mathrm{d}x\quad\text{for all }\varphi\in H^1(\Omega),$$

for it using the tirck of Minty [6], we begin by studying the limit of $\int_\Omega a(\nabla u_n)\nabla u_n\,\mathrm{d}x$. Indeed

$$\begin{aligned}\int_\Omega a(\nabla u_n)\nabla u_n\,\mathrm{d}x &=\int_\Omega f(x)u_n\,\mathrm{d}x-\int_\Omega u_n^2k(x)\,\mathrm{d}x\\ &\to\int_\Omega f(x)u\,\mathrm{d}x-\int_\Omega u^2k(x)\,\mathrm{d}x,\end{aligned}$$

because $u_n\to u$ weakly in $H^1(\Omega)$. But we know that u satisfied (11), and hence

$$\int_\Omega f(x)u\,\mathrm{d}x-\int_\Omega u^2k(x)\,\mathrm{d}x=\int_\Omega\zeta\nabla u\,\mathrm{d}x.$$

Therefore

$$\begin{aligned}\lim_{n\to+\infty}\int_\Omega a(\nabla u_n)\nabla u_n\,\mathrm{d}x &=\int_\Omega f(x)u\,\mathrm{d}x-\int_\Omega u^2k(x)\,\mathrm{d}x\\ &=\int_\Omega\zeta\nabla u\,\mathrm{d}x.\end{aligned} \tag{12}$$

Let $\varphi\in H^1(\Omega)$; it exists $(\varphi_n)_{n\in\mathbb{N}}$ such that $\varphi_n\in V_n$ for all $n\in\mathbb{N}$ and $\varphi_n\to\varphi$ in $H^1(\Omega)$ when $n\to+\infty$. We will

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pass to the limit in the term $\int_\Omega a(\nabla u_n)\nabla\varphi_n \,\mathrm{d}x$ through the monotony assumption.
Indeed,

$$0 \le \int_\Omega (a(\nabla u_n) - a(\nabla\varphi_n))(\nabla u_n - \nabla\varphi_n)\,\mathrm{d}x =$$

$$\int_\Omega a(\nabla u_n)\nabla u_n \,\mathrm{d}x - \int_\Omega a(\nabla u_n)\nabla\varphi_n \,\mathrm{d}x$$

$$- \int_\Omega a(\nabla\varphi_n)\nabla u_n \,\mathrm{d}x + \int_\Omega a(\nabla\varphi_n)\nabla\varphi_n \,\mathrm{d}x$$

$$= \quad L_{1,n} \quad - \quad L_{2,n} \quad - \quad L_{3,n} \quad + \quad L_{4,n}.$$

We saw in (12) that $L_{1,n} \to \int_\Omega \zeta\nabla u \,\mathrm{d}x$ when $n \to \infty$. We have

$$\lim_{n\to+\infty} L_{2,n} = \int_\Omega \zeta\nabla\varphi \,\mathrm{d}x.$$

Similarly,

$$\lim_{n\to+\infty} L_{3,n} = \int_\Omega a(\nabla\varphi)\nabla u \,\mathrm{d}x.$$

Finally, we also have

$$\lim_{n\to+\infty} L_{4,n} = \int_\Omega a(\nabla\varphi)\nabla\varphi \,\mathrm{d}x.$$

when $n \to +\infty$.
The passage to the limit into inequality therefore give:

$$\int_\Omega (\zeta - a(\nabla\varphi))\cdot(\nabla u - \nabla\varphi)\,\mathrm{d}x \ge 0 \text{ for all } \varphi \in H^1(\Omega).$$

We now choose astutely test function φ . We take $\varphi = u + \frac{1}{n}v$, with $v \in H^1(\Omega)$ and $n \in \mathbb{N}^*$. We thus obtained:

$$-\frac{1}{n}\int_\Omega \big(\zeta - a(\nabla u + \frac{1}{n}\nabla v)\big)\nabla v \,\mathrm{d}x \ge 0,$$

and so

$$\int_\Omega \big(\zeta - a(\nabla u + \frac{1}{n}\nabla v)\big)\nabla v \,\mathrm{d}x \le 0.$$

But $u + \frac{1}{n}v \to u$ in $H^1(\Omega)$, therefore by lemma (II.3),

$$a\big(\nabla u + \frac{1}{n}\nabla v\big) \to a(\nabla u) \text{ in } L^2(\Omega)$$

. Passing to the limit when $n \to +\infty$, then we obtained

$$\int_\Omega (\zeta - a(\nabla u))\nabla v \,\mathrm{d}x \le 0 \quad \forall v \in H^1(\Omega).$$

By linearity (can be changed v into $-v$), we have:

$$\int_\Omega (\zeta - a(\nabla u))\nabla v \,\mathrm{d}x = 0 \quad \forall v \in H^1(\Omega).$$

We deduce that

$$\int_\Omega \zeta\nabla v \,\mathrm{d}x = \int_\Omega a(\nabla u)\nabla v \,\mathrm{d}x \quad \forall v \in H^1(\Omega).$$

Hence we have to show that u is a solution of (1). ■

III. Numerical aspects and results

The artificial time discretisation associated with the functional (3) can be rewritten as follows:

$$\begin{cases} \dfrac{\partial u}{\partial t} - \operatorname{div}\big(\Psi(|\nabla u|)\nabla u\big) = f - ku & \text{in } \Omega\times(0,T) \\ \Psi(|\nabla u|)\nabla u\cdot\vec{\eta} = 0 & \text{on } \partial\Omega\times(0,T) \end{cases} \tag{13}$$

where Ψ is defined by

$$\Psi(t) = \frac{1}{1+(\frac{t}{\lambda})^2} + \frac{1}{\lambda^2} \quad \text{or } \Psi(t) = \exp\Big(-\frac{t^2}{2\lambda^2}\Big) + \frac{1}{\lambda^2}.$$

We use a finite difference scheme (see. [4]). Let us denote the space step by h supposed equal to one and the time step by Δt, we can write

$$(\nabla u)^1_{i,j} = \begin{cases} u_{i+1,j} - u_{i,j} & \text{si } i < N, \\ 0 & \text{si } i = N. \end{cases}$$

$$(\nabla u)^2_{i,j} = \begin{cases} u_{i,j+1} - u_{i,j} & \text{si } j < N, \\ 0 & \text{si } j = N. \end{cases}$$

$$|(\nabla u)_{i,j}| = \sqrt{((\nabla u)^1_{i,j})^2 + ((\nabla u)^2_{i,j})^2}.$$

For every field $p = (p_1, p_2) \in \mathbb{R}^2$, we define the discrete divergence as

$$(\operatorname{div}\mathbf{p})_{i,j} = \begin{cases} \mathbf{p}^1_{i,j} - \mathbf{p}^1_{i-1,j} & \text{si } 1 < i < N \\ \mathbf{p}^1_{i,j} & \text{si } i = 1 \\ -\mathbf{p}^1_{i-1,j} & \text{si } i = N \end{cases} + \begin{cases} \mathbf{p}^2_{i,j} - \mathbf{p}^2_{i,j-1} & \text{si } 1 < j < N \\ \mathbf{p}^2_{i,j} & \text{si } j = 1 \\ -\mathbf{p}^2_{i,j-1} & \text{si } j = N \end{cases}$$

Then we use the algorithm of chambolle that sets in [4] by

$$p^{n+1}_{i,j} = \frac{p^n_{i,j} + \tau(\nabla(\operatorname{div}p^n - f/\lambda))_{i,j}}{1 + \tau|(\nabla(\operatorname{div}p^n - f/\lambda))_{i,j}|}$$

As shown in the figures below for images 1 and 2 . The choice for our numerical tests are: for noise we use the gaussian noise (the probability density of noise k with zero mean and variance $\sigma^2 = 0,5$), the time step size $dt = 10^{-4}$ and the number of iterations is equal to 3.5×10^3. We start by the improvements tests (Fig. 3) in the restorations provided by our approach and we choose the parameter $\lambda = 1$. In the second experiment, we illustrate the difference between our proposed method and the method of the total variation and the Perona-Malik model (see Figs. 6-7) and we will show that our method is effective in reducing the staircasing effect and preserving fine details.

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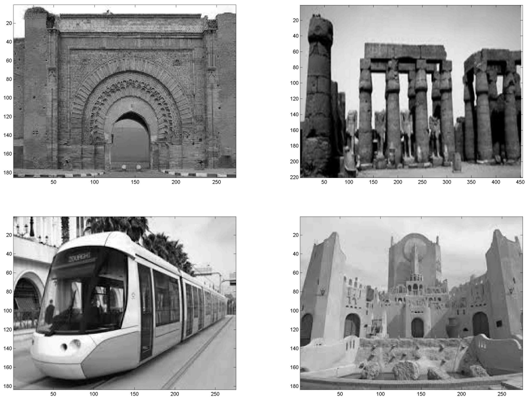


Fig. 1: Original image

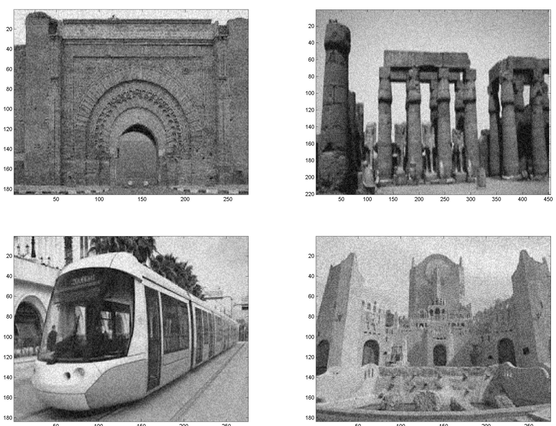


Fig. 2: Noisy image

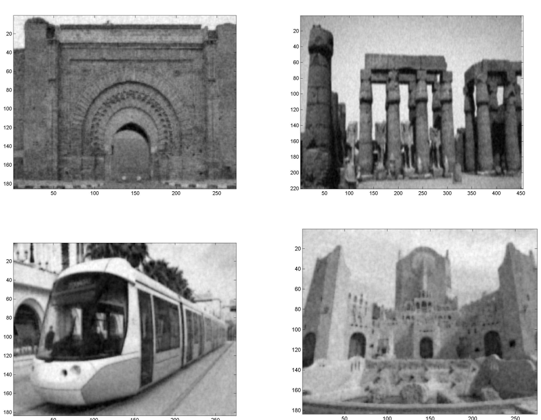
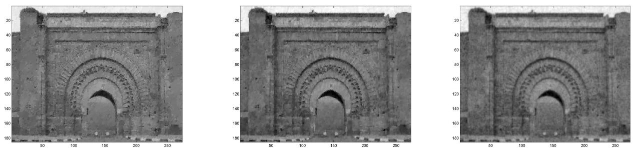
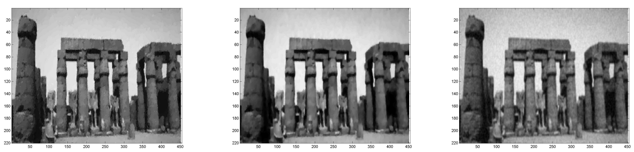


Fig. 3: Restored image by using our model



(a) (b) (c)

Fig. 4: (a) The image restored by using the Perona-Malik model (PSNR=17.9571), (b) the image restored by using the method of the total variation (PSNR=17.9714) and (c) is obtained by our model (PSNR=18.0285)

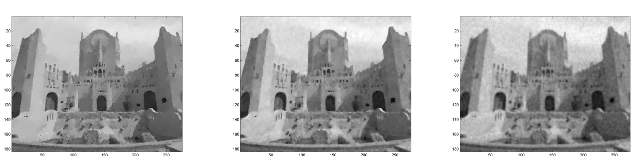


(a) (b) (c)

Fig. 5: (a) The image restored by using the Perona-Malik model (PSNR=18.2266), (b) the image restored by using the method of the total variation (PSNR=18.2295) and (c) is obtained by our model (PSNR=18,3190)

(a) (b) (c)

Fig. 6: (a) The image restored by using the Perona-Malik model (PSNR=19,6138), (b) the image restored by using the method of the total variation (PSNR=19.6166) and (c) is obtained by our model (PSNR=19,9261)



(a) (b) (c)

Fig. 7: (a) The image restored by using the Perona-Malik model (PSNR=19,8125), (b) the image restored by using the method of the total variation (PSNR=19,8469) and (c) is obtained by our model (PSNR=20,6295)

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Iteration	$PSNR_{OM}$	$PSNR_{TV}$	$PSNR_{PM}$
1	17.4285	17.3714	17.3357
50	17.7285	17.6500	17.5642
100	17.9071	17.8142	17.74258
150	17.9892	17.9285	17.8928
200	18.0285	17.9714	17.9571

TABLE I: PSNR comparisons for the three models (for image 01)

Iteration	$PSNR_{OM}$	$PSNR_{TV}$	$PSNR_{PM}$
1	17.4190	17.3523	17.3523
100	17.9952	17.8380	17.6285
200	18.2476	18.1619	17.8666
300	18.2904	18.2095	18.0666
400	18.3142	18.2285	18.1904
500	18.3190	18.2295	18.2266

TABLE II: PSNR comparisons for the three models (for image 02)

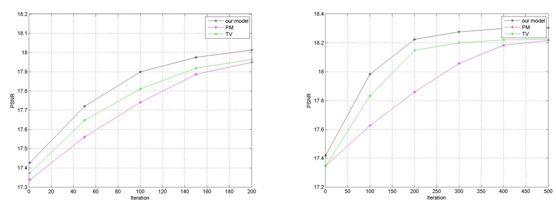

Fig. 8: The PSNR for different numbers of iterations for image 1,2

IV. Conclusion

In this paper we present results on the existence of the solution for a equation (1). these results gives us a good numerical result with a better choice of parameters in (1). Therefor shows us that our model is the best when compared with the model of Perona-Malik and the method of the total variation since the proposed model not only preserves the edges but also removes staircase during the image denoising.

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