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# IMAGE RESTORATION USING NONLINEAR ELIPTIC EQUATION

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***Abstract*—In this paper, we study the existence of solutions for nonlinear elliptic problem, in a bounded domain of $\mathbb{R}^N$, with zero Neumann boundary conditions, and give an existence theorem of weak solutions for the following equation**

$$A(u) = f(u),$$

**where** $A(u) = -\mathrm{div}\Big(g(|\nabla u|)\nabla u\Big) - \dfrac{1}{\lambda^2}\,\mathrm{div}(\nabla u)$, **and** $f \in L^2(\Omega)$**. We also give some numerical results on examples images of the application of this problème for restoration in image processing.**

***Index Terms*—Topological degree, elliptic problem, homotopy, image restoration**

## I. Introduction

Let $\Omega$ be a bounded domain in $\mathbb{R}^N$. In the classical Sobolev space $W^{1,p}(\Omega)$, A.Atlas, F.Karami, D.Meskine [1] studied the solution of the following problem:

$$u - \mathrm{div}\Big(g(|\nabla u|)\nabla u\Big) - \frac{1}{\lambda^p}\,\mathrm{div}(|\nabla u|^{p-2}\nabla u) = f.$$

where $g$ is a decreasing function defined by $g(k) = \dfrac{1}{1+(\frac{k}{\lambda})^2}$ or $g(k) = \exp\big(-\dfrac{k^2}{2\lambda^2}\big)$. We recover the linear diffusion if $g = 1$, and we remark that $g$ satisfies the following conditions.

$$\begin{cases} \lim\limits_{k\to 0} g(k) = 1, \\ \lim\limits_{k\to +\infty} g(k) = 0. \end{cases} \tag{1}$$

In this paper we study the existence of a solution for the following nonlinear problem:

$$\begin{cases} -\mathrm{div}\Big(g(|\nabla u|)\nabla u\Big) - \dfrac{1}{\lambda^2}\,\mathrm{div}(\nabla u) = f(u) \quad \text{in } \Omega, \\ \Big(g(|\nabla u|) + \dfrac{1}{\lambda^2}\Big)\nabla u \cdot \vec{\eta} = 0. \quad \text{on } \partial\Omega, \end{cases} \tag{2}$$

where $\Omega \subseteq \mathbb{R}^N$ is the bounded domain with smooth boundary $\partial\Omega$, $\lambda > 1$ be a given contrast parameter.

We assume that $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is continuous function satisfying the caratheodory conditions, and verifying also the growth restriction defined below:

$$|f(x,s)| \le d(x) + \frac{1}{2\lambda^2}|s|, \tag{3}$$

where $d \in L^2(\Omega)$ and $\lambda > 0$ is real positive constants.

Many algorithms are proposed for image processing [6], [7], [9]–[11]. In this paper, we present a new model for image restoration. The existence of solution of our PDE model is given by the compacteness methode . On the other hand we aplly our theoretical result in a noisy images (see [1], [7], [9]).

This article is organized as follows. In the next section, we give the definition of weak solution, the theorem of main result and we prove the existence of the solution of the problèm (2). And the last section is devoted to numerical aspects and results.

## II. Proposed model

Given $f \in L^2(\Omega)$, we are interested in finding weak solution of the problem (2).

We give now a definition of weak solution.

**Definition II.1.** *We say $u \in H^1(\Omega)$ is a weak solution for the problem* (2) *if for any $v \in H^1(\Omega)$ we have*

$$\int_\Omega \Big(g(|\nabla u|) + \frac{1}{\lambda^2}\Big)\nabla u \nabla v \,\mathrm{d}x = \int_\Omega f(u) v \,\mathrm{d}x. \tag{4}$$

Our main result is formulated in the following theorem.

**Theorem II.1.** *Under condition* (1) *and* (3)*, problem* (2) *has at least one solution.*

*Proof:* Let $W$ be a finite-dimensional subspace of $H^1(\Omega)$ endowed with the $H^1$-norm, and $W^*$ its dual. Define the mapping $H : W \times [0,1] \to W^*$ by

$$\langle H(u,t), v\rangle_H = \int_\Omega \Big(g(t|\nabla u|) + \frac{1}{\lambda^2}\Big)\nabla u \nabla v \,\mathrm{d}x - \int_\Omega f(tu) v \,\mathrm{d}x \tag{5}$$

for all $v \in W$. $H$ is well defined. Let us show now that $\Big\{u \in W : H(u,t) = 0, \text{ for same } t \in [0,1]\Big\} \subset \bar{B}\big(2\lambda^2\|d\|_{L^2}\big)$.


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**Lemma II.1.** *There exists $R > 0$, such that*

1)
$$\begin{cases} \forall t \in [0,1], \forall u \in V \\ H(t,u) = 0 \Rightarrow \|u\|_{H^1(\Omega)} \leq R. \end{cases}$$

2) *$H$ is bounded.*

*Proof:*
Indeed, if $H(u,t) = 0$ for same $(u,t) \in W \times [0,1]$, then

$$\begin{aligned} 0 = \langle H(u,t), u \rangle &\geq \frac{1}{\lambda^2}\|u\|^2_{H^1} - \|f(u)\|_{L^2}\|u\|_{L^2} \\ &\geq \frac{1}{\lambda^2}\|u\|_{H^1} - \|d\|_{L^2} - \frac{1}{2\lambda^2}\|u\|_{H^1}, \end{aligned}$$

which implies that

$$\|u\|_{H^1} \leq 2\lambda^2 \|d\|_{L^2}. \tag{6}$$

Consequently, for any $R > 2\lambda^2\|d\|_{L^2}$, we have

$$H(u,t) \neq 0 \text{ if } (u,t) \in \partial B^W(R) \times [0,1]. \tag{7}$$

Now, if $(u,t) \in \bar{B}^W(R) \times [0,1]$, we have

$$\begin{aligned} |\langle H(u,t), \varphi \rangle| &\leq \big(\max(1 + \frac{1}{\lambda^2}, \frac{1}{2\lambda^2})\|u\|_{H^1} + \|d\|_{L^2}\big)\|v\|_{H^1}, \\ &\leq (KR + \|d\|_{L^2})\|v\|_{H^1}, \end{aligned}$$

such that $K = \max(1 + \frac{1}{\lambda^2}, \frac{1}{2\lambda^2})$, for all $v \in H^1(\Omega)$, and hence

$$H\Big(\bar{B}^{\mathrm{W}}(R) \times [0,1]\Big) \subset \bar{B}^{\mathrm{W}^*}(KR + \|d\|_{L^2}). \tag{8}$$

■

We now show that H is continous.

**Proposition II.1.** *The mapping $H$ is continuous on*

$$\bar{B}^W(R) \times [0.1]$$

.

*Proof:* Let $(u_m, t_m) \in \bar{B}^{\mathrm{W}}(R) \times [0,1]$ converge to $(u,t)$ in $W \times [0,1]$, i.e. in $H^1 \times [0,1]$. Since $(H(u_m,t_m))$ is bounded because of (8), to prove that

$$H(u_m, t_m) \to H(u,t),$$

it is sufficient to show that $H(u,t)$ is the unique cluster point of $(H(u_m,t_m))$. Let $\ell \in \mathrm{W}^*$ be such a cluster point, still denoted by $(t_m), (u_m)$ a subsequence of $(t_m), (u_m)$ respectively such that

$$H(u_m, t_m) \to \ell \text{ in } \mathrm{W}^*.$$

Since $u_m \to u$ in $H^1(\Omega)$, it follows that $u_m \to u$ in $L^2(\Omega)$, and hence, going if necessary to a subsequence, we may assume that

$$u_m \to u \text{ a.e in } \Omega \text{ and } \exists\, H \in L^2(\Omega);\ |u_m| \leq H \text{ a.e.} \tag{9}$$

On the other hand, $\partial_i u_m \to \partial_i u$ in $L^2(\Omega)$. This implies that

$$g(t_m|\nabla u_m|) \to g(t|\nabla u|) \quad \text{ a.e in } \Omega,$$

and hence, for any $v \in W$,

$$g(t_m|\nabla u_m|)\nabla v \to g(t|\nabla u|)\nabla v$$

in $L^2(\Omega)$. We conclude that

$$\begin{aligned} &\int_\Omega \Big(g(t_m|\nabla u_m|) + \frac{1}{\lambda^2}\Big)\nabla u_m \nabla v \,\mathrm{d}x \\ &\to \int_\Omega \Big(g(t|\nabla u|) + \frac{1}{\lambda^2}\Big)\nabla u \nabla v \,\mathrm{d}x. \end{aligned}$$

For the last term,

$$f(t_m u_m) \to f(tu) \text{ a.e.},$$

by dominated convergence (from (3) and (9)) we have

$$f(t_m u_m) \to f(tu) \text{ in } L^2(\Omega),$$

and consequently

$$\int_\Omega f(t_m u_m) v \,\mathrm{d}x \to \int_\Omega f(tu) v \,\mathrm{d}x.$$

We obtain

$$\begin{aligned} &\langle H(t_m, u_m), v \rangle_H \\ &= \int_\Omega \Big(g(t_m|\nabla u_m|) + \frac{1}{\lambda^2}\Big)\nabla u_m \nabla v \,\mathrm{d}x - \int_\Omega f(t_m u_m) v \,\mathrm{d}x \\ &\to \int_\Omega \Big(g(t|\nabla u|) + \frac{1}{\lambda^2}\Big)\nabla u \nabla v \,\mathrm{d}x - \int_\Omega f(tu) v \,\mathrm{d}x \\ &= \langle H(t,u), v \rangle_H. \end{aligned}$$

Thus $\ell = H(u,t)$. ■

It is clear that

$$H : W \times [0,1] \to W^*$$

is a continuous homotopy and the existence of at least one solution of the problem (2) would follow from

$$\deg_B \Big(H(\cdot,1), B(R), 0\Big) \neq 0$$

All those proprieties allow us to apply the homotopy invariance propriety and obtain

$$\deg_B \Big(H(\cdot,1), B(R), 0\Big) = \deg_B \Big(H(\cdot,0), B(R), 0\Big). \tag{10}$$

But $H(u,0) = 0$ is equivalant to the linear problem

$$(1 + \frac{1}{\lambda^2}) \int_\Omega \nabla u \nabla v - \int_\Omega f(x,0) v = 0,$$

for all $v \in \mathrm{W}$, whose solution is unique because of the bounededness of the set of its possible solutions. Consequently,

$$\deg_B \Big(H(\cdot,0), B(R), 0\Big) = \pm 1,$$

and from (10) and the existence propriety of degree, there exists $u \in B^{\mathrm{W}}(R)$ wich satisfies

$$\begin{aligned} &\int_\Omega \Big(g(|\nabla u|) + \frac{1}{\lambda^2}\Big)\nabla u \nabla v = \int_\Omega f(u) v, \\ &\|u\|_{H^1} \leq 2\lambda^2 \|d\|_{L^2}. \end{aligned} \tag{11}$$


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for all $v \in W$.

We now show the passage to the limit.
Consider the function $a : \mathbb{R}^N \to \mathbb{R}^N$ defined by

$$a(\xi) = \Big(g(\xi) + \frac{1}{\lambda^2}\Big)\xi \quad \text{for any } \xi \in \mathbb{R}^N.$$

To prove the passage to the limit, we need the following lemma:

**Lemma II.2.** *Let $0 < \lambda \le 1$, for any $\xi, \eta \in \mathbb{R}^N$ sush that $\xi \neq \eta$ we have*

$$\big(a(\xi) - a(\eta)\big) \cdot (\xi - \eta) > 0.$$

The proof of the above lemma can be found in [?].

**Lemma II.3.** *[3]*

$$\begin{cases} \text{If } a \in C(\mathbb{R}^N, \mathbb{R}^N),\ a(\xi) \le (1 + \frac{1}{\lambda^2})\xi \quad \text{for all } \xi \in \mathbb{R}^N \\ \text{and} \\ \text{if } u_n \to u \text{ in } H^1(\Omega) \\ \text{so} \\ a(\nabla u_n) \to a(\nabla u) \text{ in } L^2(\Omega). \end{cases}$$

Lemma II.3 is proved by the dominated convergence theorem of Lebesgue. Now, it is well known that one can write $H^1(\Omega) = \overline{\bigcup_{m\ge 1} W_m}$ where $W_m \subset W_{m+1} (m \ge 1)$ and $W_m$ has dimension $m$. Consequently, given any $v \in H^1(\Omega)$, there exists a sequence $v_m$ with $v_m \in W_m$ which converges to $v$. On the other hand, by (11) applied to $W = W_m$, there exists, for each $m \ge 1$, some $u_m \in W_m$ such that

$$\int_\Omega a(\nabla u_m)\nabla\psi \, \mathrm{d}x = \int_\Omega f(u_m)\psi \, \mathrm{d}x,$$

for all $\psi \in W_m$. In particular, taking $\psi = v_m$ introduced above,

$$\begin{aligned} &\int_\Omega a(\nabla u_m)\nabla v_m \, \mathrm{d}x = \int_\Omega f(u_m) v_m \, \mathrm{d}x, \\ &\|u_m\|_{H^1(\Omega)} < 2\lambda^2 \|d\|_{L^2} \end{aligned} \tag{12}$$

for all $m \ge 1$. The estimate in (12) implies that, going if necessary to subsequences, we can assume that there exists $u \in H^1(\Omega)$ such that

$$u_m \to u \text{ weakly in } H^1(\Omega),$$

$$u_m \to u \text{ strongly in } L^2(\Omega).$$

As $\big(a(\nabla u_m)\big)_{m\in\mathbb{N}}$ is bounded in $L^2(\Omega)$, then there exists $\gamma \in L^2(\Omega)$ such that

$$a(\nabla u_m) \to \gamma \text{ weakly in } L^2(\Omega),$$

and $\nabla v_m \to \nabla v$ strongly in $L^2(\Omega)$. On the other hand, as $f(u_m) \to f(u)$ in $L^2(\Omega)$, one can let $m \to \infty$ in (12) to obtain

$$\int_\Omega \gamma \nabla v \, \mathrm{d}x = \int_\Omega f(u) v \, \mathrm{d}x \quad \text{for all } v \in H^1(\Omega). \tag{13}$$

It remains to show that

$$\int_\Omega \gamma \nabla v \, \mathrm{d}x = \int_\Omega a(\nabla u)\nabla v \, \mathrm{d}x \quad \text{for all } v \in H^1(\Omega),$$

for it using the tirck of Minty, we begin by studying the limit of $\int_\Omega a(\nabla u_m)\nabla u_m \, \mathrm{d}x$. Indeed

$$\int_\Omega a(\nabla u_m)\nabla u_m \, \mathrm{d}x = \int_\Omega f(u_m) u_m \, \mathrm{d}x \to \int_\Omega f(u) u \, \mathrm{d}x,$$

because $u_m \to u$ weakly in $H^1(\Omega)$. But we know that $u$ satisfied (13), and hence

$$\int_\Omega f(u) u \, \mathrm{d}x = \int_\Omega \gamma \nabla u \, \mathrm{d}x.$$

Therefore

$$\begin{aligned} \lim_{n\to+\infty} \int_\Omega a(\nabla u_m)\nabla u_m \, \mathrm{d}x &= \int_\Omega f(u) u \, \mathrm{d}x \\ &= \int_\Omega \gamma \nabla u \, \mathrm{d}x. \end{aligned} \tag{14}$$

Let $v \in H^1(\Omega)$; it exists $(v_m)_{m\in\mathbb{N}}$ such that $v_m \in W_m$ for all $m \in \mathbb{N}$ and $v_m \to v$ in $H^1(\Omega)$ when $m \to +\infty$. We will pass to the limit in the term $\int_\Omega a(\nabla u_m)\nabla v_m \, \mathrm{d}x$ through the monotony assumption.
Indeed,

$$\begin{aligned} 0 \le &\int_\Omega \big(a(\nabla u_m) - a(\nabla v_m)\big)(\nabla u_m - \nabla v_m) \, \mathrm{d}x = \\ &\int_\Omega a(\nabla u_m)\nabla u_m \, \mathrm{d}x - \int_\Omega a(\nabla u_m)\nabla v_m \, \mathrm{d}x \\ &- \int_\Omega a(\nabla v_m)\nabla u_m \, \mathrm{d}x + \int_\Omega a(\nabla v_m)\nabla v_m \, \mathrm{d}x \\ &= \quad S_{1,m} \quad - \quad S_{2,m} \quad - \quad S_{3,m} \quad + \quad S_{4,m}. \end{aligned}$$

We saw in (14) that $S_{1,m} \to \int_\Omega \gamma \nabla u \, \mathrm{d}x$ when $m \to \infty$. We have

$$\lim_{m\to+\infty} S_{2,m} = \int_\Omega \gamma \nabla v \, \mathrm{d}x.$$

Similarly,

$$\lim_{m\to+\infty} S_{3,m} = \int_\Omega a(\nabla v)\nabla u \, \mathrm{d}x.$$

Finally, we also have

$$\lim_{n\to+\infty} S_{4,m} = \int_\Omega a(\nabla v) \cdot \nabla v \, \mathrm{d}x.$$

when $m \to +\infty$.
The passage to the limit into inequality therefore give:

$$\int_\Omega \big(\gamma - a(\nabla v)\big)(\nabla u - \nabla v) \, \mathrm{d}x \ge 0 \text{ for all } v \in H^1(\Omega).$$

We now choose astutely test function $v$ . We take $v = u + \dfrac{1}{m}\widetilde{v}$, with $\widetilde{v} \in H^1(\Omega)$ and $m \in \mathbb{N}^*$. We thus obtained:

$$-\frac{1}{m}\int_\Omega \big(\gamma - a(\nabla u + \frac{1}{m}\nabla\widetilde{v})\big)\nabla\widetilde{v} \, \mathrm{d}x \ge 0,$$

and so


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$$\int_\Omega \big(\gamma - a(\nabla u + \frac{1}{m}\nabla\widetilde{v})\big)\nabla\widetilde{v}\,\mathrm{d}x \leq 0.$$

But $u + \frac{1}{m}\widetilde{v} \to u$ in $H^1(\Omega)$, therefore by lemma II.3,

$$a\big(\nabla u + \frac{1}{m}\nabla\widetilde{v}\big) \to a(\nabla u) \text{ in } L^2(\Omega)$$

.
Passing to the limit when $m \to +\infty$, then we obtained

$$\int_\Omega \big(\gamma - a(\nabla u)\big)\nabla\widetilde{v}\,\mathrm{d}x \leq 0 \quad \forall\widetilde{v} \in H^1(\Omega).$$

By linearity (can be changed $\widetilde{v}$ into $-\widetilde{v}$), we have:

$$\int_\Omega \big(\gamma - a(\nabla u)\big)\nabla\widetilde{v}\,\mathrm{d}x = 0 \quad \forall\widetilde{v} \in H^1(\Omega).$$

We deduce that

$$\int_\Omega \gamma\cdot\nabla\widetilde{v}\,\mathrm{d}x = \int_\Omega a(\nabla u)\nabla\widetilde{v}\,\mathrm{d}x \quad \forall\widetilde{v} \in H^1(\Omega).$$

Hence we have to show that $u$ is a solution of (2). ■

## III. Numerical aspects and results

The artificial discretisation associated with the problem (2) can be rewritten as follows:

$$\begin{cases} \dfrac{\partial u}{\partial t} - \varepsilon\mathrm{div}(\Phi(|\nabla u|)\nabla u) = f(u) \text{ in } (0,T)\times\Omega \\ \Phi(|\nabla u|)\nabla u\cdot\vec{\eta} = 0 \text{ in } (0,T)\times\partial\Omega \end{cases} \tag{15}$$

where $f$ and $\Phi$ are defined respectively by

$$f(s) = \exp\Big(-\frac{s}{\lambda}\Big) \text{ and } \Phi(t) = \frac{1}{1+(\frac{t}{\lambda})^2} + \frac{1}{\lambda^2},$$

or $\Phi(t) = \exp\Big(-\dfrac{t^2}{2\lambda^2}\Big) + \dfrac{1}{\lambda^2}$

The discretization of the Problem (15) is given by the finite difference method (see [2]). Let us $h = 1$ the space step and $\Delta t$ the time step, we can write

$$(\nabla u)^1_{i,j} = \begin{cases} u_{i+1,j} - u_{i,j} & \text{si } i < N_1, \\ 0 & \text{si } i = N_1. \end{cases}$$

$$(\nabla u)^2_{i,j} = \begin{cases} u_{i,j+1} - u_{i,j} & \text{si } j < N_1, \\ 0 & \text{si } j = N_1. \end{cases}$$

$$|(\nabla u)_{i,j}| = \sqrt{((\nabla u)^1_{i,j})^2 + ((\nabla u)^2_{i,j})^2}.$$

We can also write for every field $\mathbf{p} = (\mathbf{p}_1, \mathbf{p}_2) \in \mathbb{R}^2$, the discrete divergence approximation:

$$(\mathrm{div}\,\mathbf{p})_{i,j} = \begin{cases} \mathbf{p}^1_{i,j} - \mathbf{p}^1_{i-1,j} & \text{si } 1 < i < N_1 \\ \mathbf{p}^1_{i,j} & \text{si } i = 1 \\ -\mathbf{p}^1_{i-1,j} & \text{si } i = N_1 \end{cases} + \begin{cases} \mathbf{p}^2_{i,j} - \mathbf{p}^2_{i,j-1} & \text{si } 1 < j < N_1 \\ \mathbf{p}^2_{i,j} & \text{si } j = 1 \\ -\mathbf{p}^2_{i,j-1} & \text{si } j = N_1, \end{cases}$$

where $N_1$ is an integer greater than 2. One can write the following scheme:

$$\begin{aligned} &u^{k+1}(i,j) \\ &= u^k(i,j) + \Delta t\Big(\mathrm{div}(\Phi(|\nabla u^k(i,j)|)\nabla u^k(i,j))\Big) - f(i,j), \end{aligned}$$

where $u^k(i,j) = u(x_i, y_j, t_k),\ x_i = ih,\ y_j = jh,\ t_k = k\Delta t$ and $\Delta t = \dfrac{T}{M}$.
As shown in the figures below 1 and 2. Most algorithm parameters are chosen heuristically for the algorithms to problem their best. We choose the Gaussian noise 50%, $\Delta t = 0.3$, $\varepsilon = 2\times10^{-3}$ and the number of iterations is 700. We given firstly restore of noisy images (Figs. 2,3) by our approach and we choose the parameter $\lambda = 0.2$. In the second experiment, we give the difference between our results, the method of the total variation [2], [5]–[7] and the Perona-Malik model [1], [8], [10], [12] (see Fig 4, 6), and we give also zoom in these results (see Fig, 5, 7). We can notice from the results of PSNR in the figure 8 that our model is better then the model of Perona-Malik and the total variation method.

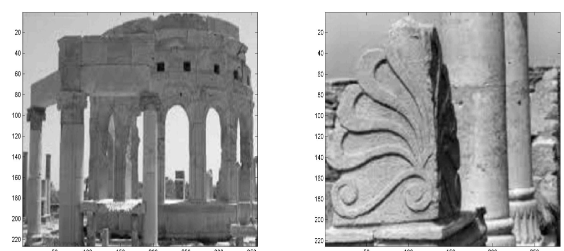

Fig. 1: Original image

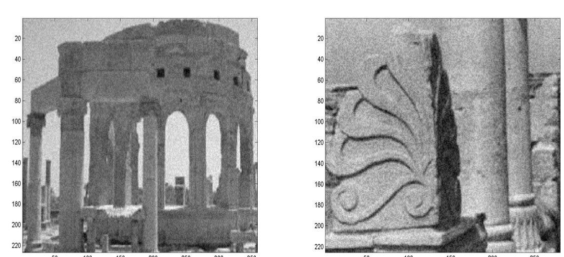

Fig. 2: Noisy image


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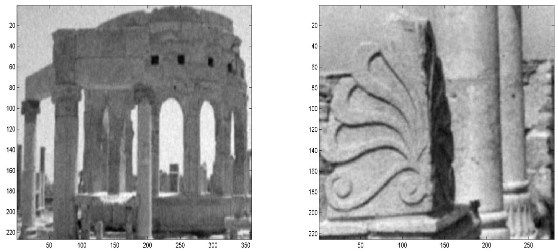
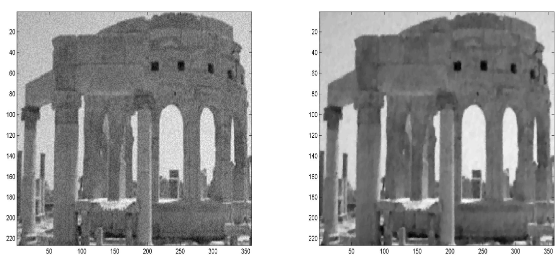
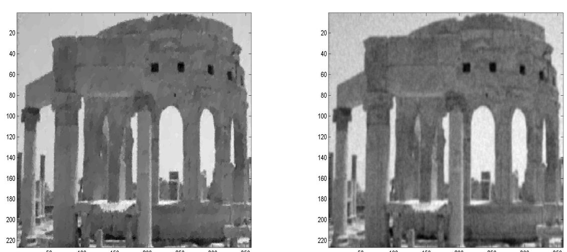


Fig. 3: Restored image by using our model with $g(k) = \dfrac{1}{1 + (\frac{k}{\lambda})^2}$

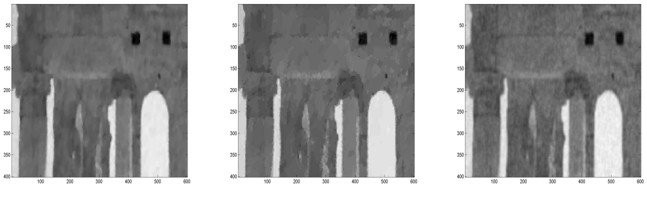


(a) (b)



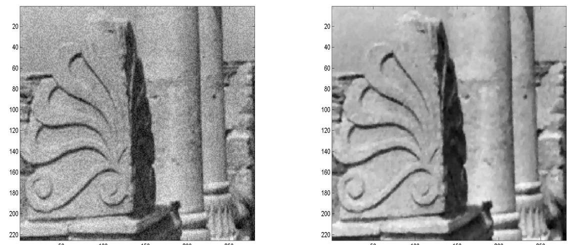
(c) (d)

Fig. 4: (a) The noisy image, (b) the image restored by using the method of the total variation, (c) the image restored by using the Perona-Malik model and (c) is obtained by our model

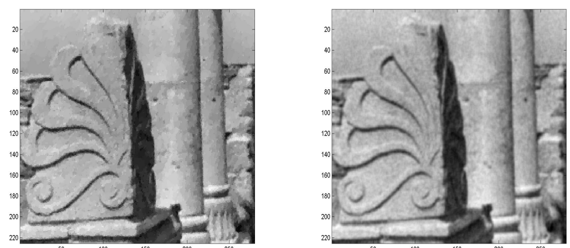


(a) (b) (c)

Fig. 5: (a) Zoom in of the image restored by TV, (b) zoom in of the image restored by PM and (c) zoom in of the our model

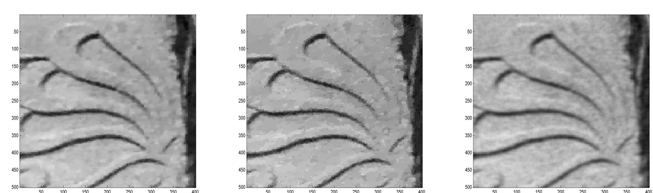


(a) (b)



(c) (d)

Fig. 6: (a) The noisy image, (b) the image restored by using the method of the total variation, (c) the image restored by using the Perona-Malik model and (d) is obtained by our model



(a) (b) (c)

Fig. 7: (a) Zoom in of the image restored by TV, (b) zoom in of the image restored by PM and (c) zoom in of the our model

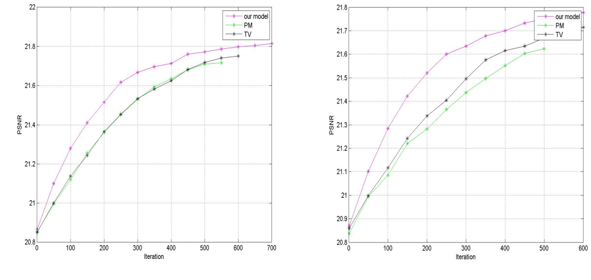



Fig. 8: The PSNR for different numbers of iterations for image 1,2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| **Iteration** | $PSNR_{OM}$ | $PSNR_{TV}$ | $PSNR_{PM}$ |
|---|---|---|---|
| 1 | 20.8668 | 20.8529 | 20.8520 |
| 100 | 21.2799 | 21.1378 | 21.1190 |
| 200 | 21.5155 | 21.3650 | 21.3607 |
| 300 | 21.6672 | 21.5327 | 21.5300 |
| 400 | 21.7134 | 21.6251 | 21.6340 |
| 500 | 21.7720 | 21.7184 | 21.7105 |
| 600 | 21.7989 | 21.7502 | 21.6962 |

TABLE I: PSNR comparisons for the three models (for image 01)

| **Iteration** | $PSNR_{OM}$ | $PSNR_{TV}$ | $PSNR_{PM}$ |
|---|---|---|---|
| 1 | 20.8694 | 20.8591 | 20.8381 |
| 100 | 21.2846 | 21.1174 | 21.0856 |
| 200 | 21.5200 | 21.3370 | 21.2816 |
| 300 | 21.6341 | 21.4951 | 21.4376 |
| 400 | 21.6781 | 21.6149 | 21.5518 |
| 500 | 21.7456 | 21.6787 | 21.6234 |
| 600 | 21.7628 | 21.7153 | / |

TABLE II: PSNR comparisons for the three models (for image 02)

## IV. Conclusion

We present in this article a novel model for image denoising (see [1]) when we shown their theoretical results. These results give the best numerical outcome with a better choice of parametres ($\Delta t$, $\lambda$,...), and we also present that our model give a better PSNR when compared with the model of Perona-Malik and the method of the total variation. This model preserve the contours and removes staircase during the image denoising.

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